



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>

EUCLID
FOR BEGINNERS
BOOKS I. AND II.

F. B. HARVEY, M. A.

E U C L I D

BOOKS I. & II.

E U C L I D

BOOKS I. & II.

LONDON : PRINTED BY
SPOTTISWOODE AND CO., NEW-STREET SQUARE
AND PARLIAMENT STREET



EUCLID

F O R B E G I N N E R S

BOOKS I. AND II.

WITH SIMPLE EXERCISES

BY THE

REV. F. B. HARVEY, M.A.

RECTOR OF CHEDDINGTON
SCHOOL INSPECTOR IN THE DIOCESE OF OXFORD
FORMERLY SECOND MASTER OF BERKHAMSTED SCHOOL



LONDON
LONGMANS, GREEN, AND CO.
1880

All rights reserved

102 111



ADVERTISEMENT

THERE are already so many and such excellent editions of Euclid that to offer another, though only of Books I. and II., may be considered both superfluous and presuming. Yet, it would be somewhat rash, on the other hand, to say that we have arrived at perfection in the matter, and that no further attempts at improvement in the publication of Euclid—for beginners, at least—need be made.

Every teacher of Euclid knows, and a long experience has confirmed the knowledge in myself, that the better the mere typical arrangement of the text, the more attractive the study of Euclid in itself is, and the quicker, and more complete, is the progress which the beginner makes. If, further, the text can be expressed more distinctly, and be brought more fully within the boy's comprehension, so that he can readily perceive what he really has to do in 'learning a Proposition,' as it is called; and be led to put a definite value upon the statement of the abstract truths he finds demonstrated, then every possible aid is afforded him in his often difficult and uninteresting work. These two points are aimed at in this edition of Euclid, as strictly a book for beginners. If the way can be smoothed thus far, the rest of the road is fully open.

As regards the first requisite, the typical arrangement of the text, it is hoped that the distinct expression, in red ink, of the particular enunciation with reference to the figure employed, and the special statement of the point or points to

be proved in the Proposition, will be found to contribute materially to the advantages spoken of. A corresponding improvement is sought in the further use, to Prop. XXVI. inclusive, of red ink, to denote the lines employed in the 'construction' of the several figures.

And with reference to the second requisite, the language of the text itself, it is believed that by a very simple deviation from the usual phraseology of the demonstration, without any sacrifice of geometrical or logical truth, a great help will be afforded to the scholar, both in learning and remembering the several Propositions.

The alteration is chiefly this:—Through the whole of the First Book there runs, as a thread, the frequent comparison of two triangles. This comparison is usually made in a manner which, to a boy, seems unnecessarily cumbrous and puzzling. Of the three parts to be taken in each of the two triangles, as equal to each other, each to each, two are first taken, and their respective equalities stated; then the third in each is taken, and their equality asserted. By this a kind of break is made in the argument, which acts as a hindrance to a learner, unimportant as it may appear to be. Now, surely, to take the three parts in each triangle, and to compare them, each with each, once for all, is just as correct, geometrically and logically, and it is certainly by far the simpler plan. A reference to the proof of Proposition V. will explain my meaning, and show, I think, the advantage claimed.

This point allowed, the application of the principle through the whole Book tends greatly to simplify it, for the purposes stated. Again, another common defect in the text is that, after the necessary comparison of two triangles has been made, while often only one of the consequences deducible is required in the argument of the Proposition, all the consequences are stated. Surely this also is cumbrous and puzzling.

For these, and other reasons which will speak for them-

selves in the several Propositions, I presume to issue this edition of Books I. and II. My original purpose would have been answered by the publication of Book I. only, but the addition of Book II., which has also its especial features, will make the whole available for some of the examinations which have to be undergone, especially since all symbols, or abbreviations, which the several examining bodies disallow, are carefully excluded. I do not suppose that I have found out the 'royal road.' I shall be more than satisfied if I do' but indicate another step in that direction.

I have not thought it necessary to introduce additional Problems, or Riders. In doing so I should be departing somewhat from the object I have chiefly in view, of preparing a book especially suitable to beginners, who, when they are sufficiently advanced for such work, will find excellent and ample material for it elsewhere. I have, however, appended some very simple Exercises, generally variations authorised by the Propositions under which they are placed. These, if they are not thought superfluous, will contribute to a more thorough understanding of the Propositions themselves, and help to train the scholar for the higher efforts of the kind which he may afterwards have to make.

Any favourable testimony that the use of this book may warrant will be appreciated, and suggestions and even hostile criticism shall have a hearty welcome, and the fullest attention.

F. B. HARVEY.

CHEDDINGTON RECTORY,
TRING.

INTRODUCTION.

THE PROPOSITIONS of Euclid are divided into two kinds, Problems and Theorems. Problems give something to be done, as the making of a Triangle. Theorems state something to be proved, as that the angles at the base of an Isosceles Triangle are equal to each other. But in Problems as well as in Theorems, argument is employed. In a Theorem the necessity of argument is apparent. In a Problem, after we have done what is required, we have to prove the accuracy of our work. Every Proposition, therefore, is to be considered as a process of reasoning. This process is carried on step by step. We start from known truths or suppositions admitted, through others, plainly flowing from or connected with them, till we arrive at the conclusion to which they unavoidably lead us. Mere assertion is allowed no place in this process. The truth of everything stated must be capable of, and have, its necessary proof. This fact the scholar must bear in mind. He must remember that to *learn a Proposition* is to get up an argument by which the statement contained in that Proposition is established. To *say*, or to write out, a *Proposition* is to reproduce that argument complete. When the scholar has grasped this idea of a Proposition, he will have made a great step, not only towards the study of Euclid successfully, but also attractively : ‘a consummation devoutly to be wished.’

From this general view of a Proposition, in Euclid, we can now proceed to a closer one.

Every Proposition contains what is called, 1. The Enunciation. 2. The Construction. 3. The Demonstration, or Proof.

INTRODUCTION.

1. The Enunciation is the original statement of the Problem, or Theorem, and it is of two kinds, general and particular.

The General Enunciation is the statement asserted generally, and printed here, and usually, in Italics.

The Particular Enunciation is the repetition of the General, asserted with reference to the particular case in which we are about to consider it. This gives us also, in distinct terms, the statement to be proved, which is printed here in red type.

2. The Construction is the addition to the lines or figures, originally given, of such other lines, or figures, as are necessary to the argument required. These Construction lines and figures are also printed here, as far as Prop. XXVI. inclusive, in red.

3. The Demonstration, or Proof, follows on the principles, above explained, of starting from truths known, or suppositions admitted, through others flowing from, or connected with, them—such as Definitions, Axioms, Postulates, Hypotheses, and other and *previous* Propositions—till we reach the evident conclusion, and this is the statement required in the Particular Enunciation to be proved. This conclusion is printed here in red type, which is thus used to show that the argument, or process of reasoning, employed is only an intervening part of the entire Proposition, and that the statement, originally made, has been demonstrated, as required.

E U C L I D.

BOOK I.

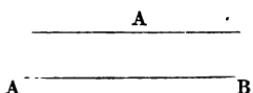
DEFINITIONS.

1.

A POINT is that which has position, but not magnitude.
A geometrical point cannot be represented without magnitude.
Its *position* is denoted by a letter, as the point A.

2.

A LINE is length without breadth.
A geometrical line cannot be represented without breadth. It
is denoted by a letter, or letters, placed on the line, or at its ex-
tremities.



3.

THE EXTREMITIES of a line are points.

4.

A STRAIGHT LINE is that which lies evenly between its
extreme points.

This is sometimes called a *right line*.

5.

A SUPERFICIES is that which has only length and
breadth.

A geometrical superficies, or, as it is sometimes called, *surface*,
cannot be represented without depth, or thickness. The shadow of

any object gives us the best idea of a superficies, or surface. A superficies is denoted by letters placed at its sides, or extremities.

6.

THE EXTREMITIES OF A SUPERFICIES are lines.

7.

A PLANE SUPERFICIES is that in which any two points being taken, the straight line between them lies wholly in that superficies.

A Plane Superficies is sometimes called 'A Plane.'
A brick has six plane superficies, or surfaces.

8.

A PLANE ANGLE is the inclination of *two lines* to each other in a Plane, which meet together, but are not in the same direction.

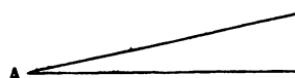
The term 'angle' in this definition denotes the *opening* which exists between two lines meeting in a point, in a plane. These 'two lines' may be straight, or curved, either or both. The only restriction is that they must meet each other, not in the same direction, in a Plane. These *Plane Angles* are not introduced in Elementary Geometry, which refers entirely to the *Plane Rectilineal Angle* spoken of in Definition 9.

9.

A PLANE RECTILINEAL ANGLE is the inclination of *two straight lines* to each other, which meet together, but are not in the same straight line.

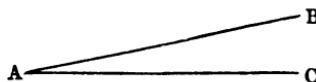
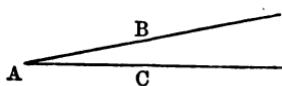
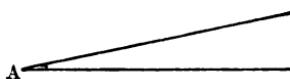
This definition is a very important one, and it must be distinctly understood.

a. The plane rectilineal angle—*angulus, a corner*—is simply the opening between two straight lines meeting at, or starting from, the same point, as A.



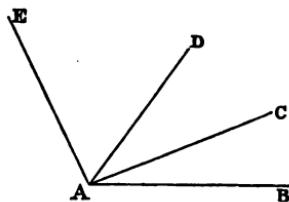
b. The point where these lines meet is called the *vertex*, and the lines themselves the *arms* of the angle.

c. An angle is denoted by a letter placed at its vertex, or by this letter with two others placed on, or at the extremities of, the lines or arms of the angle.



Thus we have the angle A , or the angle BAC . The vertex letter is always the *middle* one of the three. Hence the angle BAC is the same as the angle CAB .

d. When several lines meet and form several angles, at the same vertex, we may consider one of such angles as a part of two or more; and we may consider two or more of such angles combined as one angle.

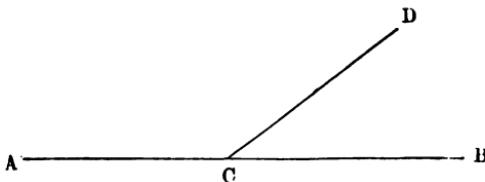


Thus we may consider the angle BAC as a part of the angle BAD , or BAE ; and we may consider the angles BAC and CAD and DAE combined as one angle BAE , &c.

e. The magnitude of an angle is explained under def. 15.

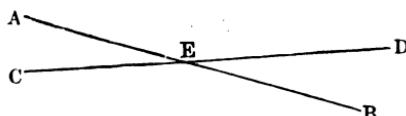
f. When a straight line meets another straight line at a point.

in the latter which is not one of its extremities, the angles thus formed are *adjacent* angles.



Thus the angles ACD and BCD are adjacent angles.

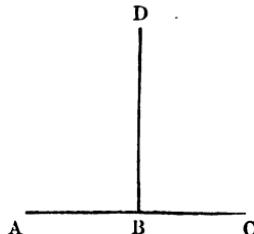
g. When two straight lines cut or intersect each other, the four angles thus formed are pairs of *vertically opposite* angles.



Thus the angles AEC and BED are vertically opposite angles ; as also are the angles AED and BEC.

10.

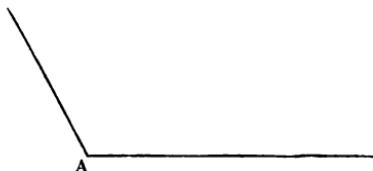
When a straight line standing upon another straight line makes the adjacent angles equal to each other, each of these angles is called a **RIGHT ANGLE**, and each straight line is said to be **PERPENDICULAR** to the other.



Thus each of the angles ABD and CBD is a right angle ; DB is perpendicular to AC, or to AB and BC ; also AB and CB are each perpendicular to BD.

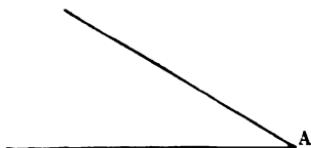
11.

AN OBTUSE ANGLE is one which is greater than a right angle.



12.

AN ACUTE ANGLE is one which is less than a right angle.



13.

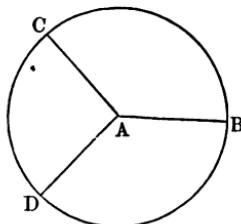
A TERM OR BOUNDARY is the extremity of anything.

14.

A FIGURE is that which is contained by one or more boundaries.

15.

A CIRCLE is a plane figure contained by one line called the **CIRCUMFERENCE**, and is such that all straight lines drawn from a certain point within it, called the **CENTRE**, to the circumference are equal to each other. Each of such equal straight lines is called a **RADIUS**.

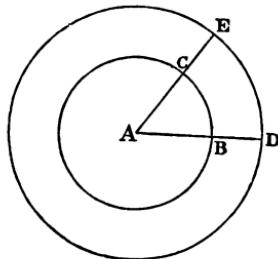


Thus BCD is a *circle* with *circumference* BCD, the centre A,

and each of the lines AB, AC, and AD is a *radius*. A part of the circumference, as BC, is called an *arc*.

Magnitude of an Angle.

1. The entire circumference of *every* circle is divided into 360 parts, called *degrees*, each deg. = 60 minutes, and each min. = 60 seconds; and an angle is said to contain as many degrees°, minutes', and seconds'', as are contained in the *arc*, or that part of the circumference which lies between the two lines forming the angle; the angular point, or *vertex*, being the *centre* of the circle.



Thus the angle BAC contains as many deg., min., and secs., as the arc BC in the smaller circle.

The angle DAE contains as many deg., min., and secs. as the arc DE in the larger circle.

Now, it can be proved that arc BC is the *same part* of its circumference that DE is of its circumference.

Therefore the angle BAC = the angle DAE, and *hence*—

2. The length of the arms of an angle makes no difference in the magnitude of that angle.

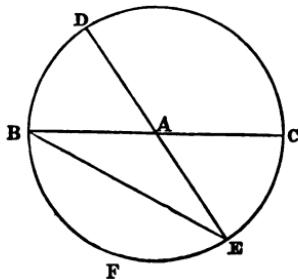
Note, also, the arms need not be of the same length. For it is plain that the angle BAC = the angle BAE, &c.

3. The arms of a Right Angle include *one-fourth* part of the circumference. A *Right Angle* contains, therefore, 90 degrees; an *obtuse* angle contains *more*, and an *acute* angle *less*, than 90 degrees.

16.

A DIAMETER OF A CIRCLE is a straight line drawn

through the *centre*, and terminated both ways by the circumference.



Thus BAC and DAE are diameters. A straight line drawn in a circle, *not through the centre*, and terminated both ways by the circumference is called a *Chord*, as the straight line BE .

17.

A SEMICIRCLE is that part of the circle which is contained by a diameter and the arc it cuts off.

In the above figure, CDB , BEC , and ECD are Semicircles.

18.

A SEGMENT OF A CIRCLE is that part of the circle which is contained by a chord and its arc.

In the above figure the chord BE divides the circle into two segments $BDCE$ and BFE .

19.

RECTILINEAL FIGURES are those which are contained by right or straight lines.

20.

A TRIANGLE is contained by three straight lines.

21.

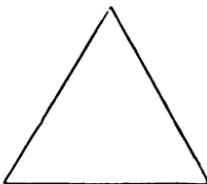
QUADRILATERAL FIGURES are contained by four straight lines.

22.

MULTILATERAL FIGURES, or POLYGONS, are contained by more than four straight lines.

23.

AN EQUILATERAL TRIANGLE has all its sides equal to each other.



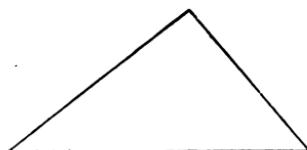
24.

AN ISOSCELES TRIANGLE has two of its sides equal to each other.



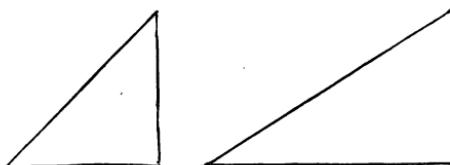
25.

A SCALENE TRIANGLE has all its sides unequal to each other.



26.

A RIGHT-ANGLED TRIANGLE has one right angle.

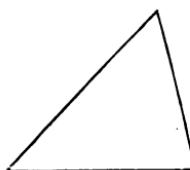


27.

AN OBTUSE-ANGLED TRIANGLE has one obtuse angle.

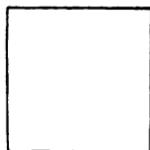
28.

AN ACUTE-ANGLED TRIANGLE has *three acute angles*.



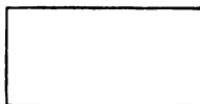
29.

A SQUARE is a Quadrilateral having all its sides equal, and all its angles right angles.



30.

AN OBLONG is a Quadrilateral which has not all its sides equal, but all its angles are right angles.



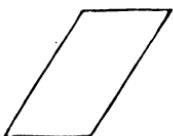
31.

A RHOMBUS is a Quadrilateral having all its sides equal, but its angles are not right angles.



32.

A RHOMBOID is a Quadrilateral having its opposite sides equal to one another, but all its sides are not equal, nor its angles right angles.



33.

All other Quadrilaterals besides these are called TRAPEZIUMS.

Some writers on Mensuration of Surfaces speak of a *Trapezoid* as a Quadrilateral having *one* pair of opposite sides parallel, and consider the *Trapezium* as having neither pair of opposite sides parallel. This is not in accordance with Euclid's language in Book I. Prop. 35, where a Quadrilateral with one pair of opposite sides parallel is called a Trapezium.

This thirty-third def. is limited by Def. 34, which is usually appended as a Note to the Enunciation of Prop. 34, Book I.

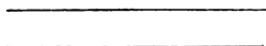
34.

A PARALLELOGRAM is a Quadrilateral of which the *opposite sides* are parallel ; and the *diagonal*, or diameter, is the straight line joining two of its opposite angles.

The Square, Oblong, Rhombus, and Rhomboid are each of them Parallelograms, as this definition shows. For the terms Rhombus and Rhomboid that of Parallelogram is often used ; and for Oblong the term Rectangle.

35.

PARALLEL STRAIGHT LINES are such as are in the same plane, and which, being continually produced, never meet.



POSTULATES.

1.

Let it be granted that a straight line may be drawn from any one point to any other point.

2.

That a terminated straight line may be produced to any length in a straight line.

3.

That a circle may be described from any centre, at any distance from that centre.

The Postulates are 'Requests' that Euclid makes for certain things to be allowed as permissible in the study of Geometry. They are but three: 1. The *drawing* of a straight line from any one point to any other. 2. The *producing*, to any length, of a straight line already drawn. 3. The *describing* of a circle from any *centre* with any *radius*.

AXIOMS.

1.

Things which are equal to the same thing are equal to one another.

2.

If equals be added to equals, the wholes are equal.

3.

If equals be taken from equals, the remainders are equal.

4.

If equals be added to unequals, the wholes are unequal.

5.

If equals be taken from unequals, the remainders are unequal.

6.

Things which are double of the same are equal to one another.

7.

Things which are halves of the same are equal to one another.

8.

Magnitudes which coincide with one another—that is, which fill exactly the same space—are equal to one another.

9.

The whole is greater than its part.

10.

Two straight lines cannot enclose space.

11.

All right angles are equal to one another.

12.

If a straight line meets two straight lines so as to make the two interior angles on the same side of it, taken together, less than two right angles, these straight lines being continually produced shall at length meet upon that side on which are the angles which are less than two right angles.

The Axioms are ‘Common Notions,’ or self-evident Truths. To them Euclid, on this ground, claims assent. Axioms 10, 11, and 12 are considered by some to be of the nature of Postulates rather than Axioms. This, however, is a distinction the consideration of which the beginner in Euclid may postpone.

HINTS TO THE LEARNER.

1. Make the figures of a good size, and as accurately as possible. A well-drawn figure is of great value towards the understanding of the Proposition.
 2. Do not *copy* the figures of any Proposition, but draw them, step by step, as directed in the 'Construction.'
 3. Remember that in the 'Proof' of any Proposition Euclid employs those Propositions only which are *previous* to the one under consideration. He never expects you to have a knowledge beyond what you ought thus to have already acquired.
-

EXPLANATION OF TERMS.

A COROLLARY is a Theorem, or Problem, which arises easily and directly from the Proposition to which it is attached.

HYPOTHESIS is a *supposition* assumed, for the time, to be true.

Q. E. F. stand for *Quod erat faciendum*, meaning *which was to be done*. They stand at the end of *Problems*.

Q. E. D. stand for *Quod erat demonstrandum*, meaning *which was to be demonstrated* or proved. They stand at the end of *Theorems*.

The following abbreviations are used :

| | | | |
|--------------|---------------|--------------|-------------|
| <i>ax.</i> | axiom. | <i>ext.</i> | exterior. |
| <i>alt.</i> | alternate. | <i>fig.</i> | figure. |
| <i>comp.</i> | complement. | <i>hyp.</i> | hypothesis. |
| <i>cons.</i> | construction. | <i>int.</i> | interior. |
| <i>cor.</i> | corollary. | <i>opp.</i> | opposite. |
| <i>def.</i> | definition. | <i>post.</i> | postulate. |



E U C L I D.

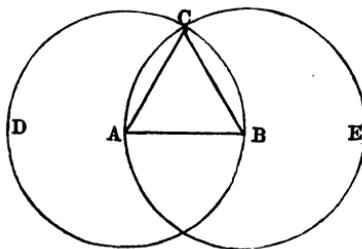
BOOK I.

PROP. I. PROBLEM.

To describe an equilateral triangle upon a given finite straight line.

Let AB be the given straight line.

It is required to describe on AB an equilateral triangle.



CONSTRUCTION.—1. From centre A, with distance AB, describe the circle BCD (post. 3).

2. From centre B, with distance BA, describe the circle ACE.

3. From point C, where the circles cut each other, draw CA and CB (post. 1).

Then, it is to be proved that

ABC is an equilateral triangle described upon AB.

PROOF.—*Because A is the centre of the circle BCD, therefore AB=AC (def. 15).*

Similarly, because B is the centre of the circle ACE, therefore BA=BC.

But it has been proved that BA=AC, therefore AC=BC (ax. 1), and therefore AB, BC, and CA = each other.

Therefore, it is proved, as required, that

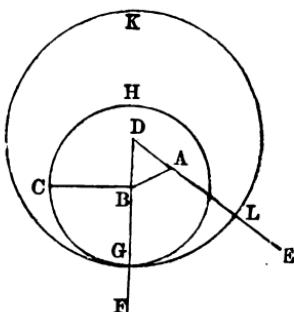
ABC is an equilateral triangle, described upon AB.

Q. E. F.

PROP. II. PROBLEM.

From a given point to draw a straight line equal to a given straight line.

Let A be the given point, and BC the given straight line.
It is required to draw from A a straight line equal to BC.



CONSTRUCTION.—1. From A to B draw the straight line AB (post. 1).

2. Upon AB describe the equilateral triangle BDA (I. 1).
3. Produce DA and DB to the points E and F (post. 2).
4. From centre B, with distance BC, describe the circle CHG, cutting DF in G (post. 3).
5. From centre D, with distance DG, describe the circle GKL cutting DE in L.

Then, it is to be proved that

AL is the line drawn from $A=BC$.

PROOF.—Because B is the centre of the circle CHG, therefore $BG=BC$ (def. 15).

Similarly, because D is the centre of the circle GKL, therefore $DG=DL$.

But in the lines DG and DL we have $DB=DA$ (cons.), therefore $BG=AL$ (ax. 3).

Also it has been shown that $BG=BC$; therefore $AL=BC$ (ax. 1).

Therefore, it is proved, as required, that

AL is the line drawn from $A=BC$.

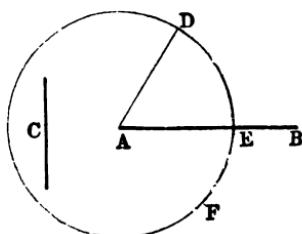
Q. E. F.

PROP. III. PROBLEM.

From the greater of two given straight lines to cut off a part equal to the less.

Let AB and C be the two given straight lines, of which AB is the greater.

It is required to cut off from AB, the greater, a part equal to C, the less.



CONSTRUCTION.—1. From A draw AD=C (I. 2).

2. From centre A, with distance AD, describe the circle DEF, cutting AB in E (post. 3).

Then, it is to be proved that

AE is cut off from AB=C.

PROOF.—*Because A is the centre of the circle DEF, therefore AE=AD (def. 15).*

But AD=C (cons.); therefore AE=C (ax. 1).

Therefore, it is proved, as required, that

AE is cut off from AB=C.

Q. E. F.

Exercises.

1. Describe an equilateral triangle on a given straight line MN, with the vertex, O, below MN.

2. Prove Prop. II. when A is joined to C, instead of to B.

N.B.—The Exercises given in the following pages are not necessarily connected with the Proposition under which they are placed. But they are strictly confined to that or to previous Propositions.

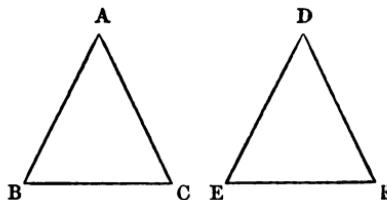
PROP. IV. THEOREM.

If two triangles have two sides of the one equal to two sides of the other, each to each, and have also the angles contained by those sides equal to each other, then they shall have their bases, or third sides, equal; and the two triangles shall be equal; and their other angles shall be equal, each to each, viz., those to which the equal sides are opposite.

In the triangles ABC and DEF let the sides AB and AC, and their angle BAC, in the former=the sides DE and DF, and their angle EDF, in the latter, each to each.

Then it is to be proved that

1. The base BC = the base EF.
2. The triangle ABC = the triangle DEF.
3. The angle ABC = the angle DEF.
4. The angle ACB = the angle DFE.



PROOF.—If the triangle ABC be placed upon the triangle DEF so that the point A is on the point D, and the side AB on the side DE, then, because AB=DE (hyp.), therefore the point B shall coincide with the point E, and the side AC shall coincide with the side DF.

Next, because the side AB coincides with the side DE, and because the angle BAC = the angle EDF (hyp.), therefore the side AC shall fall on the side DF, and, because the point A coincides with the point D, and AC=DF, (hyp.), therefore the point C shall coincide with the point F.

But we have also seen that the point B coincides with the point E; *therefore* the whole base BC shall coincide with the whole base EF.

For, if the point B coincides with the point E, *and* the point C coincides with the point F, *then, if* the whole base BC does *not* coincide with the whole base EF, we have two straight lines enclosing a space, which is impossible (ax. 10).

Therefore,

1. The base BC coincides with the base EF.
2. The triangle ABC coincides with the triangle DEF.
3. The angle ABC coincides with the angle DEF.
4. The angle ACB coincides with the angle DFE.

And therefore, it is proved (ax. 8), as required, that

1. The base BC = the base EF.
2. The triangle ABC = the triangle DEF.
3. The angle ABC = the angle DEF.
4. The angle ACB = the angle DFE.

Wherefore,

If two triangles, &c.

Q. E. D.

Exercises.

1. Given the straight lines AB and CD, of which AB is the greater; it is required to produce CD to make it = AB.
2. Prove Prop. IV. when the triangle DEF is applied to the triangle ABC.

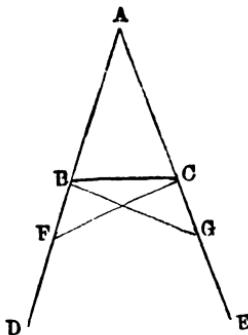
PROP. V. THEOREM.

The angles at the base of an isosceles triangle are equal to each other, and, if the equal sides are produced, the angles upon the other side of the base are also equal to each other.

Let ABC be an isosceles triangle with the side AB = the side AC, and let AB and AC be produced to D and E respectively.

Then it is to be proved that

1. The angles ABC and ACB, at the base = each other.
2. The angles DBC and ECB, upon the other side of the base, also = each other.



CONSTRUCTION.—1. In BD take any point F.

2. From AE cut off AG = AF (I. 3), and join BG and CF.

PROOF.—*Because* in the triangles FAC and GAB we have the sides FA and AC, and their angle FAC, in the former = the sides GA and AB, and their angle GAB, in the latter, each to each (hyp. and cons.), *therefore* the base CF = the base BG, the angle ACF = the angle ABG, and the angle AFC = the angle AGB (I. 4), i.e. the angle BFC = the angle CGB (note 2 def. 15).

Next, *because* in the triangles BCF and CBG we have

the sides BF and FC, and their angle BFC, in the former = the sides CG and GB, and their angle CGB, in the latter, each to each (cons. and proof above), therefore the angle BCF = the angle CBG, and the angle FBC = the angle GCB (I. 4), i.e. the angle DBC = the angle ECB (note 2 def. 15), and these are the angles upon the other side of the base.

Further, because the angle ABG = the angle ACF, and the angle CBG = the angle BCF, as already proved, therefore if the angle CBG be taken from the angle ABG, and the angle BCF be taken from the angle ACF, then the remaining angle ABC = the remaining angle ACB (ax. 3), and these are the angles at the base.

Therefore, it is proved, as required, that

1. The angles ABC and ACB, at the base = each other, and
2. The angles DBC and ECB, upon the other side of the base. also = each other.

Wherefore,

The angles at the base of an isosceles triangle, &c.

Q. E. D.

Cor. Every equilateral triangle is also equiangular.

NOTE.—It may assist the scholar, in learning this proposition, to observe that in the first step in the proof the larger pair of triangles, FAC and GAB, and in the second step the smaller pair, BCF and CBG, are taken.

He will also notice that the equality of the angles ‘on the other side of the base’ is proved, before the equality of those ‘at the base’ is demonstrated.

Exercise.

Given an isosceles triangle BAC with the vertical angle at A bisected by AD drawn to BC; prove that AD is perpendicular to BC.

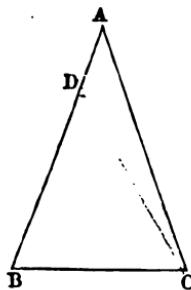
PROP. VI. THEOREM.

If two angles of a triangle are equal to each other, then the sides also which subtend, or are opposite to, the equal angles, are equal to each other.

Let ABC be a triangle having the angle ABC = the angle ACB :

Then it is to be proved that

The side AB = the side AC.



CONSTRUCTION.—Suppose that AB is *greater* than AC.

From AB cut off a part DB = AC (I. 3) and join CD.

PROOF.—*Because* in the triangles DBC and ACB we have the sides DB and BC and their angle DBC, in the former = the sides AC and CB, and their angle ACB, in the latter, each to each (hyp. and cons.), *therefore* the triangle DBC = the triangle ACB (I. 4), i.e. a part = the whole, which is absurd (ax. 9).

Therefore, the supposition that AB is *greater* than AC is absurd.

Similarly the supposition that AB is *less* than AC might be shown to be absurd.

Therefore, it is proved, as required, that

The side AB = the side AC.

Wherefore,

If two angles of a triangle, &c.

Q. E. D.

Cor.—Every equiangular triangle is also equilateral.

N.B.—The proof here given is an instance of what is called *Indirect Demonstration*. This means, the truth of the statement asserted is only proved by showing that absurdity follows from an argument based on the supposition that such statement is not true.

The scholar must be careful to notice that the term ‘absurd,’ which is found in this method of proof, does not apply to the *entire* demonstration, but only to the *hypothesis* on which it is based. The chain of argument beyond this, in these cases, is strictly correct.

Exercises.

1. Prove the above Proposition on the supposition that AC is greater than AB.
2. If a straight line DA be drawn at right angles to another straight line BC from its middle point D, *prove*, if BA and CA be joined, that BA = CA.

PROP. VII. THEOREM.

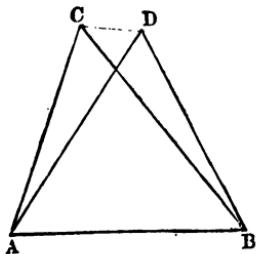
Upon the same base, and on the same side of it, there cannot be two triangles that have their sides terminated in one extremity of the base equal to each other, and likewise those which are terminated in the other extremity of the base equal to each other.

Suppose it possible that on the same base, A.B, and on the same side of it—e.g. above it—there can be two triangles, ACB and ADB, having the sides AC and AD terminated in one extremity of the base, A, = each other, and having also the sides BC and BD terminated in the other extremity of the base, B, = each other;

Then this supposition will present itself in three cases.

CASE I.—Where the vertex of each triangle falls *without* the other, as in the following figure.

CONSTRUCTION.—Join the vertices CD.



PROOF.—*Because* in the triangle ACD the side AC = the side AD (hyp.), therefore the angle ACD = the angle ADC (I. 5).

But the angle ACD is greater than the angle BCD (ax. 9); therefore the angle ADC is also greater than the angle BCD.

Again, because the angle BDC is greater than the angle ADC (ax. 9), therefore the angle BDC is *much greater* than the angle BCD.

Next, because in the triangle BDC the side BD = the side BC (hyp.), therefore the angle BDC = the angle BCD (I. 5).

But we have just proved that the angle BDC is “*much greater*” than the angle BCD.

Therefore, on the above supposition, the angle BDC is *both equal to, and greater than*, the angle BCD, which is absurd.

Therefore the above supposition is false as referred to CASE I., where the vertex of one triangle falls *without* the other.

We now pass on to consider this same supposition under

CASE II.—Where the vertex of one triangle, D, falls *within the other*, as in the following figure.

CONSTRUCTION.—1. Join the vertices CD.

2. Produce AC and AD to E and F respectively.

PROOF.—*Because* in the triangle ACD, the side AC = the side AD (hyp.) and these sides are produced to E and F respectively, *therefore* the angle ECD = the angle FDC (I. 5).

But the angle ECD is greater than the angle BCD (ax. 9); *therefore* the angle FDC is also greater than the angle BCD.

Again, *because* the angle BDC is greater than the angle FDC (ax. 9), *therefore* the angle BDC is *much greater* than the angle BCD.

Next, *because* in the triangle BDC, the side BD = the side BC (hyp.), *therefore* the angle BDC = the angle BCD (I. 5).

But we have just proved that the angle BDC is "*much greater*" than the angle BCD.

Therefore, on the above supposition, the angle BDC is *both equal to, and greater than*, the angle BCD, which is absurd.

Therefore the above supposition is false as referred to CASE II., where the vertex of one triangle falls within the other.

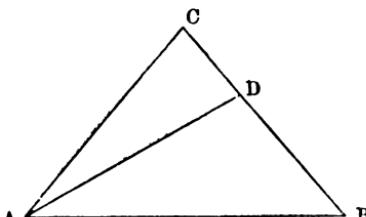
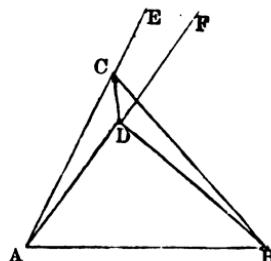
CASE III.—Where the vertex of one triangle falls *on* a side of the other, as in the following figure.

In this case it is evident that if one pair of the sides which are terminated in one extremity of the base, e.g. AC and AD, terminated in A, equal each other, then the other pair, BC and BD, terminated in B, *cannot* be equal to each other, as the Proposition requires, and therefore this case is to be dismissed.

Wherefore,

Upon the same base, &c.

Q. E. D.



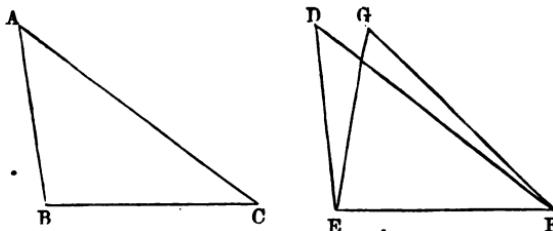
PROP. VIII. THEOREM.

If two triangles have the three sides of the one equal to the three sides of the other, each to each, then the angle which is contained by any two sides of the one triangle shall be equal to the angle contained by the two sides, equal to them, of the other.

In the triangles ABC and DEF, let the sides AB, BC, and CA in the former = the sides DE, EF, and FD in the latter, each to each.

Then it is to be proved that

1. The angle BAC = the angle EDF.
2. The angle ABC = the angle DEF.
3. The angle ACB = the angle DFE.



PROOF.—1. If the triangle ABC be placed upon the triangle DEF, so that the point B is on the point E, and the side BC upon the side EF, then, because BC = EF (hyp.), therefore the point C will coincide with the point F, and because the point B coincides with the point E, and the point C with the point F, therefore the side BC coincides with the side EF.

Next, because the side BC coincides with the side EF therefore the sides BA and AC shall coincide with the sides ED and DF respectively.

For, if BC coincides with EF, and then BA and AC do not coincide with ED and DF, suppose that BA and AC have another direction, as EG and GF, then, if this be true, we shall have upon the same base EF, and upon the same side of it, two triangles in a manner which is impossible (I. 7).

Therefore, if BC coincides with EF, then BA and AC must coincide with ED and DF, and the angle BAC will coincide with, and equal, the angle EDF (ax. 8).

Therefore, it is proved, as required, that

1. The angle BAC = the angle EDF.

Similarly,

2. The angle ABC = the angle DEF.
3. The angle ACB = the angle DFE.

Wherefore,

If two triangles, &c.

Q. E. D.

N.B.—The equality of the two triangles, in every respect, follows from this Proposition, as it does from Prop. IV.

Exercises.

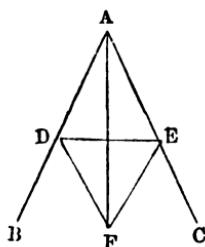
1. In the fig. Prop. I. if E be the point where the circles intersect below AB, and AE and BE be joined; prove that the angle ACB = the angle AEB.
2. If BC be the base of an isosceles triangle with vertical angle at A; prove that if a line AD bisects the base BC, it bisects also the vertical angle BAC.

PROP. IX. PROBLEM.

To bisect a given rectilineal angle, i.e. to divide it into two equal parts.

Let BAC be the given rectilineal angle:

It is required to bisect it.



CONSTRUCTION.—1. In AB take any point, D .

2. From AC cut off $AE = AD$ (I. 3).

3. On DE construct an equilateral triangle DEF (I. 1).

4. Join AF .

Then it is to be proved that

The rectilineal angle BAC is bisected by the line AF .

PROOF.—*Because* in the two triangles DAF and EAF , we have the three sides DA , AF , and FD in the former = the three sides EA , AF , and FE , in the latter, each to each (cons.), therefore the angle DAF = the angle EAF (I. 8), i.e. the angle BAF = the angle CAF (note 2 def. 15).

Therefore, it is proved, as required, that

The rectilineal angle BAC is bisected by the line AF .

Q. E. F.

Exercise.

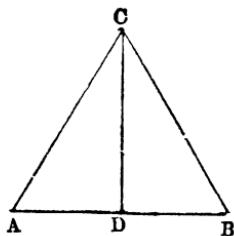
In the fig. Prop. I. if E be the point where the circles intersect below AB , and AE and BE be joined; prove that AB bisects the angle CBE .

PROP. X. PROBLEM.

To bisect a given finite straight line, i.e. to divide it into two equal parts.

Let A.B be the given finite straight line :

It is required to bisect it.



CONSTRUCTION.—1. On AB construct an equilateral triangle ABC (I. 1).

2. Bisect the angle ACB by CD, cutting AB in D (I. 9.)

Then it is to be proved that

The given straight line AB is bisected in D.

PROOF.—*Because* in the triangles ACD and BCD we have the sides AC and CD, and their angle ACD, in the former = the sides BC and CD, and their angle BCD, in the latter, each to each (cons.), *therefore* the base AD = the base BD (I. 4).

Therefore, it is proved, as required, that

The given straight line AB is bisected in D.

Q. E. F.

Exercise.

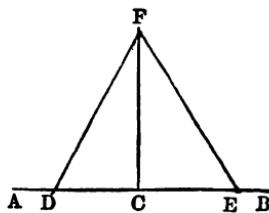
Given an isosceles triangle BAC with the vertical angle at A bisected by AD, meeting BC in D; prove that BC is bisected in D.

PROP. XI. PROBLEM.

To draw a straight line at right angles to a given straight line, from a given point in the same.

Let AB be the given straight line, and C the given point in it :

It is required to draw from C a straight line at right angles to AB.



CONSTRUCTION.—1. Take any point D in AC.

2. Make CE = CD (I. 3).

3. Upon DE construct an equilateral triangle DFE (I. 1);

4. Join FC.

Then it is to be proved that

The straight line FC is drawn from C at right angles to AB.

PROOF.—*Because* in the triangles DCF and ECF we have the three sides DC, CF, and FD, in the former = the three sides EC, CF, and FE, in the latter, each to each (cons.), therefore the angle DCF = the angle ECF (I. 8), and these are *adjacent* angles, and therefore right angles (def. 10).

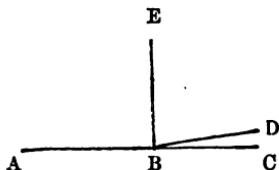
Therefore, it is proved, as required, that

The straight line FC is drawn from C at right angles to AB.

Q. E. F.

COROLLARY.

By help of this Problem it may be demonstrated that
Two straight lines cannot have a common segment.



Suppose it possible that ABC and ABD are two straight lines, with a common segment, or part, AB.

CONSTRUCTION.—From B draw BE at right angles to AB (I. 11).

PROOF.—Because ABC is a straight line (hyp.) with BE perp. to it (cons.), therefore the angle ABE = the angle EBC (def. 10); and because ABD is a straight line (hyp.) with BE perp. to it (cons.), therefore the angle ABE = the angle EBD (def. 10); and therefore the angle EBD = the angle EBC (ax. 1), i.e. the less = the greater, which is absurd (ax. 9).

The same result would follow for any other position of ABC and ABD, with AB as a part of each.

Therefore, it is proved, as required, that

Two straight lines cannot have a common segment.

Q. E. D.

Exercises.

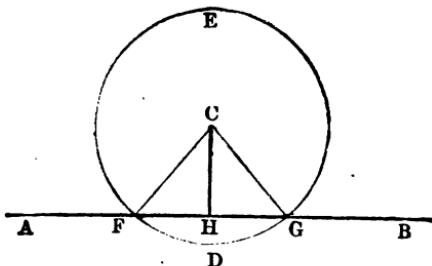
1. Two points, A and B, are given *above* a straight line CD; find a point E in the straight line CD, so that if AE and BE be joined, AE = BE.
2. Find the same when A is *above*, and B *below*, the straight line CD, but the line joining A and B *not* perpendicular to CD.
3. Find the same when A is *above*, and B *in*, the straight line CD.

PROP. XII. PROBLEM.

To draw a straight line at right angles to a given straight line of unlimited length, from a given point without it.

Let AB be the given straight line of unlimited length, or that which may be produced to any distance, both ways; and C the given point without it.

It is required to draw from C a straight line at right angles to AB.



CONSTRUCTION.—1. Take any point, D, upon the other side of AB.

2. From centre C with distance CD describe the circle EFG, cutting AB in F and G (post. 3).

3. Bisect FG in H (I. 10).

4. Join CF, CH, and CG.

Then it is to be proved that

The straight line CH is drawn from C at right angles to AB.

PROOF.—*Because* in the triangles FHC and GHC, we have the three sides FH, HC, and CF, in the former = the three sides GH, HC, and CG, in the latter, each to each

(cons.), therefore the angle FHC = the angle GHC (I. 8); and these are *adjacent angles*, and therefore right angles (def. 10).

Therefore, it is proved, as required, that

The straight line CH is drawn from C at right angles to AB.

Q. E. F.

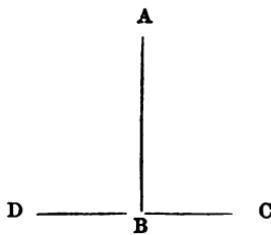
Exercises.

1. Let ABCD be a square and AC and BD its diagonals; prove that AC = BD.
2. Let ABCD be a rhomboid; prove that its *opposite angles* = each other.
3. Let ABCD be a rhombus with diagonal AC; prove that the angle DAC = the angle BAC, and that the angle BCA = the angle DCA.

PROP. XIII. THEOREM.

The angles which one straight line makes with another upon one side of it are either two right angles, or are together equal to two right angles.

CASE I.—When the angles are equal to each other.



Let the straight line AB make with the straight line CD, on one side of it, the angles ABC and ABD = each other.

Then it is to be proved that

The angles ABC and ABD are two right angles.

PROOF.—Because the angle ABC = the angle ABD (hyp.), therefore each of them is a right angle (def. 10).

Therefore, it is proved, as required, that

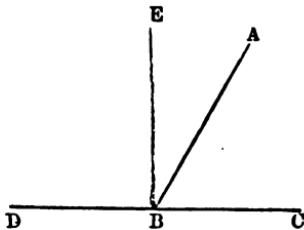
The angles ABC and ABD are two right angles.

CASE II.—When the angles are not equal to each other.

Let the straight line AB make with CD, on one side of it, the angles ABC and ABD not equal to each other.

Then it is to be proved that

The angles ABC and ABD are together = two right angles.



CONSTRUCTION.—Draw BE at right angles to CD (I. 11).

PROOF.—The angles DBE and CBE = two right angles (def. 10).

Now,¹ the angle EBC = the angles ABC and ABE, and, adding angle EBD to each of these, then the angles EBC and EBD = the angles ABC, ABE, and EBD (ax. 2).

Again,² the angle ABD = the angles ABE and EBD, and, adding angle ABC to each of these, then the angles ABD and ABC = the angles ABC, ABE, and EBD (ax. 2).

But, as above, the angles EBC and EBD = the same angles ABC, ABE, and EBD; therefore the angles EBC and EBD = the angles ABD and ABC (ax. 1).

But the angles EBC and EBD = two right angles (cons.); therefore the angles ABD and ABC = two right angles (ax. 1).

Therefore, it is proved, as required, that

The angles ABC and ABD are together equal to two right angles.

Wherefore,

The angles which one straight line makes with another, &c.

Q. E. D.

¹ I.e. the double angle on the right side of the figure.

² I.e. the double angle on the left side of the figure.

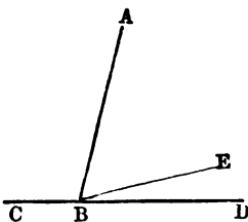
PROP. XIV. THEOREM.

If at a point in a straight line, two other straight lines, upon opposite sides of it, make the adjacent angles together equal to two right angles, then these two straight lines shall be in one and the same straight line.

At the point B, in the straight line AB, let the two straight lines BC and BD, on opposite sides of AB, make the adjacent angles ABC and ABD together = two right angles.

Then it is to be proved that

The two straight lines BC and BD are in one and the same straight line.



CONSTRUCTION.—*If* BC and BD are not in the same straight line, then suppose that BC and BE are, i.e. that CBE is one straight line.

PROOF.—Because CBE is a straight line, and AB is another line falling upon it in B, therefore the angles ABC and ABE together = two right angles (I. 13).

But the angles ABC and ABD together = two right angles (hyp.), and therefore the angles ABC and ABD = the angles ABC and ABE (ax. 1). Take away the common angle ABC, and then the angle ABE = the angle ABD (ax. 3), i.e. a part = the whole, which is absurd (ax. 9).

Therefore the supposition that BC and BE are in the same straight line is false.

Similarly it can be proved that only BC and BD are in the same straight line.

Therefore, it is proved, as required, that

The two straight lines BC and BD are in one and the same straight line.

Wherefore,

If at a point in a straight line, &c.



Q. E. D.

PROP. XV. THEOREM.

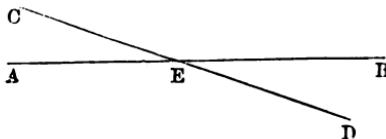
If two straight lines cut one another, then the vertical, or opposite, angles are equal to each other.

Let the two straight lines AB and CD cut each other in the point E.

Then it is to be proved that

The angle AEC = the angle BED, and
Similarly,

The angle CEB = the angle AED.



PROOF.—Because the angles AEC and AED = two right angles, and because also the angles AED and DEB = two right angles (I. 13); therefore the angles AEC and AED = the angles AED and DEB (ax. 1); take away the common angle AED from each, then the remaining angle AEC = the remaining angle DEB.

Therefore, it is proved, as required, that

The angle AEC = the angle BED.

Similarly,

The angle CEB = the angle AED.

Wherefore,

If two straight lines cut one another, &c.

Q. E. D.

COROLLARIES.

1. If two straight lines cut one another, the four angles they make at the point where they cut, are together equal to four right angles.

2. If any number of straight lines cut one another, all the angles made by them where they cut, are together equal to four right angles.

Exercises.

1. Prove in Prop. XV. that the angle CEB = the angle AED.

2. Prove each of the above Corollaries.

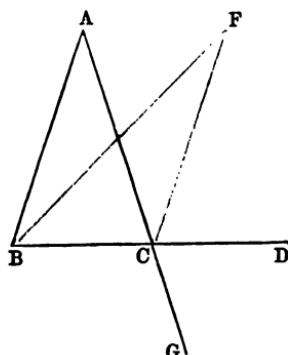
PROP. XVI. · THEOREM.

If one side of a triangle be produced, the exterior angle is greater than either of the interior opposite angles.

Let ABC be a triangle, and let the side BC be produced to D.

Then it is to be proved that

The exterior angle ACD shall be greater than either the interior opposite angles ABC or BAC.*



CONSTRUCTION.—1. Bisect AC in E by the line BE (I. 10).

2. Produce BE to F (post. 2) making $EF = BE$ (I. 3), and join FC.

PROOF.—Because in the triangles AEB and FEC, we have the sides AE, and EB, and their angle AEB, in the former = the sides CE and EF, and their angle FEC, in the latter, each to each (cons. and I. 15), therefore the angle BAE = the angle ECF (I. 4), i.e. the angle BAC = the angle ACF (note 2 def. 15).

But the angle ACD is greater than the angle ACF (ax. 9), i.e. the exterior angle ACD is greater than the interior opposite angle BAC.

Similarly,

If we bisect BC and produce AC to G, &c., as before, then the angle BCG would be proved to be greater than the angle ABC.

But the angle BCG = the angle ACD (I. 15), therefore the exterior angle ACD is greater than the interior opposite angle ABC.

Therefore, it is proved, as required, that

The exterior angle ACD is greater than either the int. opp. angles ABC or BAC.

Wherefore,

If one side of a triangle be produced, &c.

Q. E. D.

* With ACD as the *exterior angle*, there are three *interior angles*, BAC, ABC, and ACB. But angle ACB is *interior and adjacent*, whilst the angles BAC and ABC are *interior and opposite*; and it is of *these* the Proposition speaks.

Exercises.

1. Prove, in Prop. XVI., that when BC is bisected, and AC produced to G, the angle BCG is *greater* than the angle ABC.
2. If ABC be a triangle with the side BC produced to D, and with the exterior angle ACD *bisected* by CF, and the interior adjacent angle ACB *bisected* by CG; prove that the angle GCF = a right angle.

PROP. XVII. THEOREM.

Any two angles of a triangle are together less than two right angles.

Let ABC be a triangle.

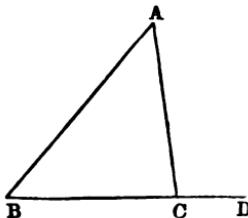
Then it is to be proved that

The angles ABC and ACB are together less than two right angles.

Similarly,

The angles ABC and BAC are together less than two right angles;

And the angles ACB and BAC are together less than two right angles.



CONSTRUCTION.—Produce the side BC to D.

PROOF.—*Because* the exterior angle ACD is greater than the interior and opposite angle ABC (I. 16), add to each of these the angle ACB, then the angles ACD and ACB are greater than the angles ABC and ACB (ax. 4).

But the angles ACD and ACB = two right angles (I. 13), and

Therefore, it is proved, as required, that

The angles ABC and ACB are less than two right angles.

Similarly,

The angles ABC and BAC are together less than two right angles.

And the angles ACB and BAC are together less than two right angles.

Wherefore,

Any two angles of a triangle, &c.

Q. E. D.

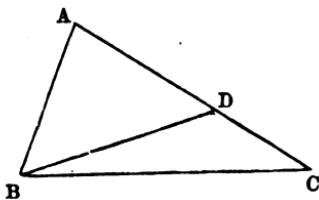
PROP. XVIII. THEOREM.

The angle which is opposite to the greater side in any triangle is greater than the angle which is opposite to the less.

Let ABC be a triangle, of which the side AC is greater than the side AB.

Then it is to be proved that

The angle ABC is greater than the angle ACB.



CONSTRUCTION.—Because AC is greater than AB (hyp.), make $AD = AB$ (I. 3), and join BD.

PROOF.—Now, the exterior angle ADB is greater than the interior and opposite angle BCD (I. 16), i.e. the angle BCA (note 2 def. 15); and the angle ABD = the angle ADB (I. 5); therefore the angle ABD is also greater than the angle BCA.

Again, the angle ABC is greater than the angle ABD (ax. 9).

Therefore, it is proved, as required, that

The angle ABC is greater than the angle ACB.

Wherefore,

In any triangle, &c.

Q. E. D.

Exercise.

Prove, in Prop. XVII., that, as stated, the angles ABC and BAC are together less than two right angles.

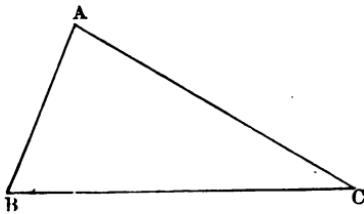
PROP. XIX. THEOREM.

The side which is opposite to the greater angle in any triangle is greater than the side which is opposite to the less.

Let ABC be a triangle, of which the angle ABC is greater than the angle ACB.

Then it is to be proved that

The side AC is greater than the side AB.



PROOF.—*If* the side AC is *not* greater than the side AB, *then* it is plain that AC must be *either = or less than* AB.

Now, if the side AC = AB, *then* the angle ABC = the angle ACB (I. 5); *but* the angle ABC is greater than the angle ACB (hyp.), *therefore* the side AC is *not =* the side AB.

Next, if the side AC is less than the side AB, *then* the angle ABC is less than the angle ACB (I. 18); *but* the angle ABC is greater than the angle ACB (hyp.), *therefore* the side AC is *not less than* the side AB.

Since, then, the side AC is *neither equal to nor less than* the side AB,

Therefore, it is proved, as required, that

The side AC is greater than the side AB.

Wherefore,

In any triangle, &c.

Q. E. D.

PROP. XX. THEOREM.

- Any two sides of a triangle are together greater than the third side.

Let ABC be a triangle.

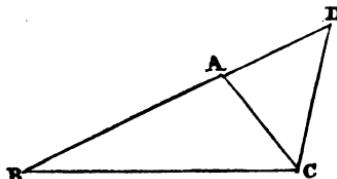
Then it is to be proved that

The sides BA and AC are together greater than the side BC.

Similarly,

The sides AB and BC are together greater than the side AC.

And the sides BC and CA are together greater than the side AB.



CONSTRUCTION.--Produce BA to D, making $AD = AC$ (I. 3), and join DC.

PROOF.--Because the side $AD =$ the side AC (cons.), therefore the angle $ADC =$ the angle ACD (I. 5); but the angle BCD is greater than the angle ACD (ax. 9); therefore the angle BCD is greater than the angle ADC , i.e. the angle BDC (note 2 def. 15), and therefore the side BD is greater than the side BC (I. 19).

But the side $BD =$ the sides BA and AC (cons.)

Therefore, it is proved, as required, that

The sides BA and AC are together greater than the side BC .

Similarly,

The sides AB and BC are together greater than the side AC , and

The sides BC and CA are together greater than the side AB .

Wherefore,

Any two sides of a triangle, &c.

Q. E. D.

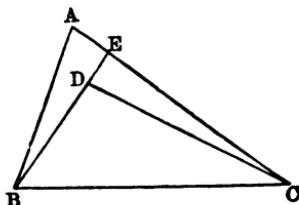
PROP. XXI. THEOREM.

If from the ends of the side of a triangle two straight lines be drawn to a point within the triangle, these shall be together less than the other two sides of the triangle, but they shall contain a greater angle.

Let ABC be a triangle, and from B and C, the ends of the side BC, let the two straight lines BD and CD be drawn to a point D within the triangle.

Then it is to be proved that

1. The two lines BD and DC are together less than the two sides BA and AC; but
2. The angle BDC is greater than the angle BAC.



CONSTRUCTION.—Produce BD to meet AC in E.

PROOF.—Because in the triangle BAE, the sides BA and AE are together greater than BE (I. 20), add to each of these EC; then the sides BA, AE, and EC, i.e. the sides BA and AC, are together greater than the sides BE and EC (ax. 4).

Again, because in the triangle CED the sides CE and ED are together greater than CD (I. 20), add to each of these DB, then the sides CE, ED, and DB, i.e. the sides CE and EB, are together greater than the sides CD and DB (ax. 4).

But we have already proved that the sides BA and AC are together greater than the sides CE and EB; and therefore the sides BA and AC are together much greater than the ~~sides CD and BD~~.

Therefore, it is proved, as required, that

1. The two lines BD and DC are together less than the two sides BA and AC.

Next, because the exterior angle BDC of the triangle DCE is greater than the interior and opposite angle CED (I. 16), i.e. the angle CEB (note 2 def. 15), and also the exterior angle CEB is greater than the interior and opposite angle BAE (I. 16), i.e. the angle BAC,

Therefore, it is proved, as required, that

2. The angle BDC is greater than the angle BAC.

Wherefore,

If from the ends, &c.

Q. E. D.

Exercises.

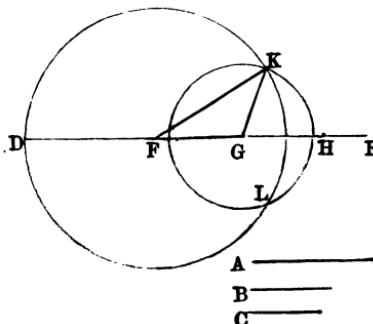
1. If from a point A two straight lines, AB and AC, be let fall upon another straight line ED, the line AB being perpendicular to ED; prove that AB subtends, or is opposite to, an acute angle.
2. Prove, in Prop. XX., that, as stated, the sides AB and BC are together greater than the side AC.
3. If a point A and a straight line BC be given; prove that the shortest line that can be drawn from A to BC, say AD, is perpendicular to BC.

PROP. XXII. PROBLEM.

To make a triangle of which the sides shall be equal to three given straight lines, any two of which are together greater than the third.

Let A, B, and C be three given lines any two of which are together greater than the third.

It is required to make a triangle having its sides = A, B, and C, each to each.



CONSTRUCTION.—1. Take a straight line DE, terminated at D, but unlimited towards E.

2. In this line make $DF = A$, make $FG = B$, and $GH = C$ (I. 3).

3. From centre F with radius FD describe the circle DKL.

4. From centre G with radius GH describe the circle HKL.

5. Join KF and KG.

Then it is to be proved that

The triangle KFG is the triangle required.

PROOF.—Because F is the centre of the circle DKL, therefore $FK = FD$ (def. 15), and $FD = A$ (cons.), therefore $FK = A$ (ax. 1).

Again, because G is the centre of the circle HKL, therefore $GK = GH$ (def. 15), and $GH = C$ (cons.), therefore $GK = C$ (ax. 1); also $FG = B$ (cons.), therefore, the triangle KFG has its three sides KF , FG , and GK = the three given lines A, B, and C, each to each.

Therefore, it is proved, as required, that

The triangle KFG is the triangle required.

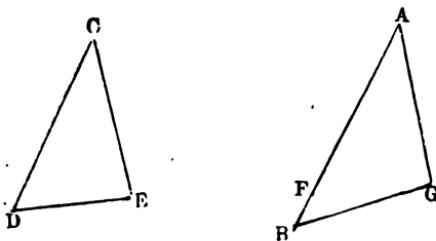
Q. E. F.

PROP. XXIII. PROBLEM.

At a given point in a given straight line, to make a rectilineal angle equal to a given rectilineal angle.

Let AB be the given straight line, A the given point in it, and DCE the given rectilineal angle.

It is required to make at A a rectilineal angle = the rectilineal angle DCE.



CONSTRUCTION.—1. In CD and CE take any points D and E and join DE.

2. Make the triangle FAG = the triangle DCE, so that the three sides FA, AG, and GF = the sides DC, CE, and ED, each to each (I. 22).

Then it is to be proved that

The angle FAG is the angle required.

PROOF.—*Because* in the triangles FAG and DCE we have the three sides FA, AG, and GF in the one = the three sides DC, CE, and ED in the other, each to each (cons.), *therefore* the angle FAG = the angle DCE (I. 8).

Therefore, it is proved that

The angle FAG is the angle required.

Q. E. F.

Exercise.

If ABC be a triangle with the side AB greater than AC; prove that the difference between AB and AC is less than the side BC.

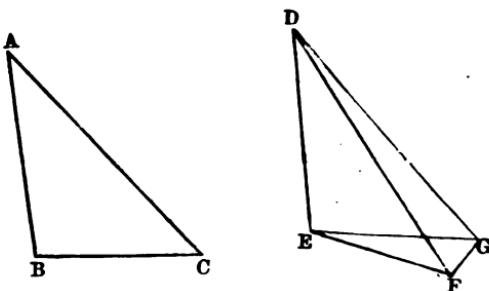
PROP. XXIV. THEOREM.

If two triangles have two sides of the one equal to two sides of the other, each to each, but the angles contained by these sides unequal, then their bases or third sides shall be unequal, and the base of that triangle which has the greater angle shall be greater than the base of the other.

Let ABC and DEF be two triangles, with the sides AB and AC in the former = the sides DE and DF, in the latter, each to each, but with the angle BAC greater than the angle EDF.

Then it is to be proved that

The base BC is greater than the base EF.



CONSTRUCTION.—Of the two sides DE and DF, let DE be not greater than DF.

1. At the point D in the straight line ED make the angle EDG = the angle BAC (I. 23).

2. Make DG = AC or DF, and join EG and GF.

PROOF.—*Because* in the triangles BAC and EDG we have the sides BA and AC and their angle BAC, in the former = the sides ED and DG and their angle EDG in the latter, each to each (hyp. and cons.), *therefore* the base BC = the base EG (I. 4).

Again, because DG = DF (cons.), *therefore* the angle DGF = the angle DFG (I. 5).

But the angle DGF is greater than the angle EGF (ax. 9); *therefore* the angle DFG is also greater than the angle EGF.

Again, because the angle EFG is greater than the angle DFG (ax. 9), therefore the angle EFG is much greater than the angle EGF, and therefore the side EG is greater than the side EF (I. 19).

But it has been shown that $EG = BC$.

Therefore, it is proved, as required, that

The base BC is greater than the base EF.

Wherefore,

If two triangles, &c.

Q. E. D.

N.B.—Compare this Proposition with Prop. IV.

Exercises.

1. Prove that from the same point A above a given straight line CD, only one perpendicular AB can be drawn to CD.

2. Prove that from the same point A in a given straight line CD, only one perpendicular AB can be drawn to CD.

3. If ABC be a triangle with CD drawn from the vertical angle C, bisecting AB in D; prove that AC and CB are together greater than twice CD.

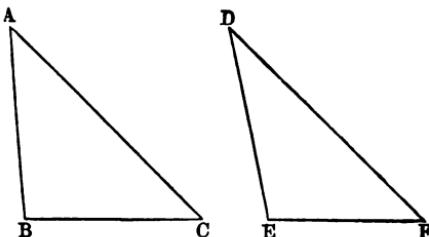
PROP. XXV. THEOREM.

If two triangles have two sides of the one equal to two sides of the other, each to each, but their bases or third sides unequal to each other, then the angles contained by the equal sides in each triangle shall be unequal, and the angle contained by the two sides of that triangle which has the greater base shall be greater than the angle contained by the two sides equal to them of the other.

Let ABC and DEF be two triangles with the sides AB and AC in the former = the sides DE and DF in the latter, each to each, but with the base BC greater than the base EF.

Then it is to be proved that

The angle BAC is greater than the angle EDF.



PROOF.—*If the angle BAC be not greater than the angle EDF, then it is plain that the angle BAC must be either =, or less than, the angle EDF.*

Now, if the angle BAC = the angle EDF, then the base BC = the base EF (I. 4); but the base BC is greater than the base EF (hyp.); therefore the angle BAC is not equal to the angle EDF.

Next, if the angle BAC is less than the angle EDF, then the base BC would be less than the base EF (I. 24); but the base BC is greater than the base EF (hyp.); therefore the angle BAC is not less than the angle EDF.

Since, then, the angle BAC is *neither equal to, nor less than*, the angle EDF,

Therefore, it is proved, as required, that

The angle BAC is greater than the angle EDF.

Wherefore,

If two triangles, &c.

Q. E. D.

N.B.—Compare this Proposition with Prop. VIII.

Exercises.

1. If ABC be a triangle, and D a point within it, with straight lines joining DA, DB, and DC; *prove* that the lines DA, DB, and DC are together *less* than the sides of the triangle AB, BC, and CA.
2. If ABCD be any quadrilateral, with AC and BD its diagonals; *prove* that the four sides AB, BC, CD, and DA are together *greater* than the diagonals AC and BD together.

PROP. XXVI. THEOREM.

If two triangles have two angles of the one equal to two angles of the other, each to each, and one side equal to one side, namely, either the sides adjacent to the equal angles, in each, or the sides opposite to equal angles in each, then the other two sides shall be equal, each to each, and also the third angle of the one triangle shall be equal to the third angle of the other.

Let $\triangle ABC$, $\triangle DEF$ be two triangles which have the angles ABC and ACB in the former = the angles DEF and DFE in the latter, each to each; and let the side *adjacent* to the equal angles in the former triangle = the side *adjacent* to the equal angles in the latter; or let the equal sides be those *opposite* to equal angles in each triangle.

Then

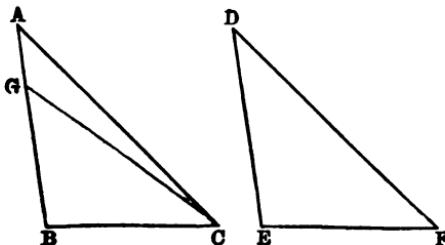
1. The other two sides in the former triangle = the other two sides in the latter; and
2. The third angle in the former triangle = the third angle in the latter.

CASE I.

First, let the given equal sides be those *adjacent* to the equal angles in each triangle, viz. BC and EF .

Then it is to be proved that

1. The other two sides AB and AC in the former triangle = the other two sides DE and DF in the latter; and
2. The third angle BAC in the former triangle = the third angle EDF in the latter.



CONSTRUCTION.—*If AB be not equal to DE , then it is*

plain that one of them must be greater than the other. Let AB be the greater; from it cut off $BG = DE$ (I. 3), and join GC .

PROOF.—*Because* in the triangles GBC and DEF we have the sides GB and BC , and their angle GBC , in the former = the sides DE and EF , and their angle DEF , in the latter, each to each (cons. and hyp.), *therefore* the angle GCB = the angle DFE (I. 4); *but* the angle DFE = the angle ACB (hyp.); *therefore* the angle GCB = the angle ACB (ax. 1), i.e. a part equals the whole, which is absurd (ax. 9). *Therefore* the supposition that AB is *not equal* to DE is erroneous; *consequently* $AB = DE$.

Again: *because* in the triangles ABC and DEF we now have the sides AB and BC , and their angle ABC , in the former = the sides DE and EF , and their angle DEF , in the latter, each to each (hyp.), *therefore* the base AC = the base DF , *and* the angle BAC = the angle EDF (I. 4).

Therefore, when the equal sides are *adjacent* to the equal angles in each triangle, it is proved, as required, that

1. The other two sides BA and AC in the former triangle = the other two sides DE and EF in the latter; and that
2. The third angle BAC in the former triangle = the third angle EDF in the latter.

Q. E. D.

Exercises.

Prove the above Case—

1. By supposing DE to be greater than AB .
2. By supposing AC to be greater than DF .
3. By supposing DF to be greater than AC .

CASE II.

Secondly, let the given equal sides be those *opposite* to equal angles in each triangle, viz., AB equal to DE.

Then it is to be proved that

1. The other two sides BC and CA in the former triangle = the other two sides EF and FD in the latter; and
2. The third angle BAC of the former triangle = the third angle EDF of the latter.

CONSTRUCTION.—*If BC be not equal to EF, then it is plain that one of them must be greater than the other.* Let BC

be the greater; from it cut off $BH = EF$ (I. 3), and join AH.

PROOF.—*Because* in the triangles ABH and DEF we have the sides AB and BH, and their angle ABH, in the former = the sides DE and EF, and their angle

DEF, in the latter, each to each (cons. and hyp.), therefore the angle BHA = the angle EFD (I. 4); but the angle EFD = the angle BCA (hyp.); therefore the angle BHA = the angle BCA (ax. 1); but the angle BHA is greater than the angle HCA (I. 16), i.e. the angle BCA (note 2 def. 15). Hence the angle BHA is both =, and greater than, the angle BCA, which is absurd. Therefore the supposition that BC is not equal to EF is erroneous; consequently $BC = EF$.

Again. *Because* in the triangles ABC and DEF we now have the sides AB and BC, and their angle ABC, in the former = the sides DE and EF, and their angle DEF, in the latter, each to each, therefore the base AC = the base DF, and the angle BAC = the angle EDF (I. 4).

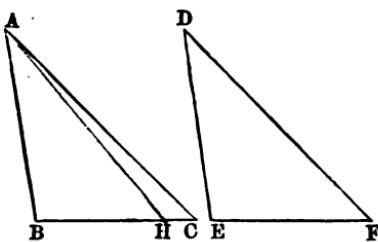
Therefore, when the equal sides are *opposite* to the equal angles in each triangle, it is proved, as required, that

1. The other two sides BC and CA in the former triangle = the other two sides EF and FD in the latter; and
2. The third angle BAC of the former triangle = the third angle EDF of the latter.

Wherefore,

If two triangles, &c.

Q. E. D.



ANGLES MADE BY INTERSECTING LINES.

When one straight line falls upon two other straight lines, whether these two lines are parallel to each other or not, the angles formed at the several points of intersection have certain technical names; and these angles and their names must be thoroughly understood before proceeding to the next Propositions.

This is easily done by reference to the accompanying figures.

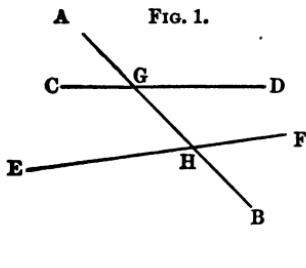


FIG. 1.

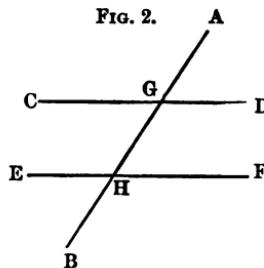


FIG. 2.

In fig. 1. Let the straight line AB cut the straight lines CD and EF , which are *not parallel* to each other, in G and H .

In fig. 2. Let the straight line AB cut the straight lines lines CD and EF , which are *parallel* to each other, in G and H .

Then, in *each* figure,

1. The angles AGC and AGD *above* CD are *exterior*, or *outside*, angles.

Also the angles BHE and BHF *below* EF are *exterior*, or *outside*, angles.

2. The angles CGH and DGH *below* CD are *interior*, or *inside*, angles.

Also the angles EHG and FHG *above* EF are *interior*, or *inside*, angles.

3. The angles CGH and GHF are one pair of *alternate* angles.

And the angles DGH and GHE are also a pair of *alternate* angles.

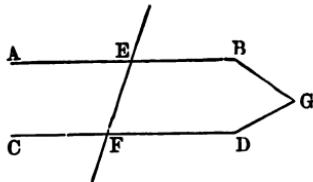
PROP. XXVII. THEOREM.

If a straight line falling upon two other straight lines make the alternate angles equal to one another, these two straight lines are parallel to each other.

Let the straight line EF falling upon the two straight lines AB and CD make the alternate angles AEF and EFD = each other.

Then it is to be proved that

The straight lines AB and CD are parallel.



CONSTRUCTION.—*If AB is not parallel to CD, then AB and CD being produced will meet, either towards A and C, or towards B and D.*

Let them be produced towards B and D, and meet in G.

PROOF.—*Because AB and CD are produced to meet in G (hyp.), therefore EGF is a triangle, and the exterior angle AEF is greater than the interior opposite angle EFG (I. 16).*

But also the angle AEF = the angle EFD (hyp.), i.e. the angle EFG (note 2 def. 15), which is absurd.

Therefore the supposition that AB and CD meet when produced towards B and D is erroneous.

Similarly, the supposition that AB and CD meet if produced towards A and C, is erroneous.

Therefore, it is proved, as required, that

The straight lines AB and CD are parallel (def. 35).

Wherefore,

If a straight line, &c.

Q. E. D.

N.B.—In the above figure the scholar must not be puzzled because EGF is called a *triangle*. The reason why it is not triangle in appearance is because the size of the page does not permit the lines AB and CD to meet to form an actual triangle.

Exercises.

1. Prove Prop. XXVI. Case ii. by taking AC and DF as the pair of equal sides *opposite* to the equal angles in each triangle.
2. If AD bisecting the vertical angle A of a triangle ABC be perpendicular to the base BC; *prove* that the triangle is isosceles.
3. If A and B are two points, A above and B below a straight line CD, and CD bisects AB in E; *prove* that if AF and BG be drawn to CD at right angles to it, AF = BG.

PROP. XXVIII. THEOREM.

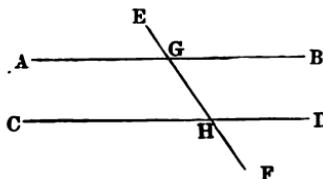
If a straight line falling upon two other straight lines make the exterior angle equal to the interior and opposite angle on the same side of the line; or make the two interior angles on the same side together equal to two right angles, these two straight lines are parallel to each other.

Let the straight line EF falling upon the two straight lines AB and CD make

1. Either the exterior angle EGB = the interior and opposite angle GHD on the same side of EF, or
2. The two interior angles BGH and GHD on the same side of EF = two right angles.

Then, in either case, it is to be proved that

The straight lines AB and CD are parallel.



PROOF.—1. Because the angle EGB = the angle GHD (hyp.), and because the angle EGB = the angle AGH (I. 15), therefore the angle AGH = the angle GHD (ax. 1); and these are alternate angles.

Therefore, it is proved, as first required, that

The straight lines AB and CD are parallel.

2. Because the angles BGH and GHD = two right angles (hyp.), and because the angles AGH and BGH = two right angles (I. 13), therefore the angles AGH and BGH = the angles BGH and GHD (ax. 1); remove the common angle BGH, and then the remaining angle AGH = the remaining angle GHD (ax. 3); and these are alternate angles.

Therefore, it is proved, as required in the second place,
that

The straight lines AB and CD are parallel.

Wherefore,

If a straight line, &c.

Q. E. D.

Exercises.

1. Prove the above Proposition by taking the angles on the *left* of the cutting line EF, viz. the exterior angle EGA and the interior and opposite angle GHC, *and* the two interior angles AGH and GHC.
2. If any angle BAC be bisected by a straight line AD, and *any* point E be taken in AD; prove that if straight lines EF and EG be drawn to AB and AC respectively, perpendicular to AD, then $EF = EG$.
3. If in a triangle ABC, with vertex C, the sides AC and BC be bisected at right angles by DF and EF respectively meeting in F; prove that FG drawn at right angles to the third side AB will bisect it in G.

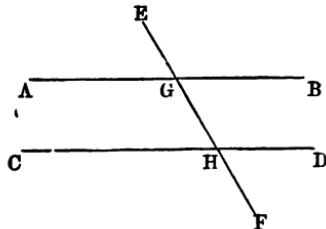
PROP. XXIX. THEOREM.

If a straight line fall on two parallel straight lines it makes the alternate angles equal to one another; the exterior angle equal to the interior and opposite angle on the same side; and also the two interior angles on the same side taken together equal to two right angles.

Let the straight line EF fall on the two parallel lines AB and CD.

Then it is to be proved that

1. The alternate angle AGH = the alternate angle GHD.
2. The exterior angle EGB = the interior and opposite angle GHD.
3. The two interior angles BGH and GHD = two right angles.



PROOF.—1. *If the angle AGH is not equal to the angle GHD, then one of them must be greater than the other.*

Suppose that the angle AGH is greater than the angle GHD, and to each of them add the angle BGH; then the angles AGH and BGH are greater than the angles GHD and BGH (ax. 4).

But the angles AGH and BGH = two right angles (I. 13); therefore the angles BGH and GHD must be less than two right angles; and therefore the straight lines AB and CD will meet if produced (ax. 12). But they never can meet when produced, because by hypothesis they are parallel

[def. 35). Therefore the supposition that the angle AGH is greater than the angle GHD is erroneous. Similarly, the supposition that the angle AGH is less than the angle GHD might be shewn to be erroneous. Consequently, the angle AGH = the angle GHD.

Therefore, it is proved, as required, that

1. The alternate angle AGH = the alternate angle GHD.
2. Next : Because the angle EGB = the angle AGH (I. 15), and because, as it has just been proved, the angle AGH = the angle GHD, therefore the angle EGB = the angle GHD (ax. 1).

Therefore, it is proved, as required, that

2. Next : Because the angle EGB = the interior and opposite angle GHD.
3. Further : Because, as we have just proved, the angle EGB = the angle GHD, add to each of them the angle BGH, then the angles EGB and BGH = the angles BGH and GHD (ax. 2). But the angles EGB and BGH = two right angles (I. 13), therefore the angles BGH and GHD = two right angles (ax. 1).

Therefore, it is proved, as required, that

3. The two interior angles BGH and GHD = two right angles.

Wherefore,

If a straight line, &c.

Q. E. D.

Exercise.

1. Prove the above Proposition by taking
 1. The alternate angles BGH and GHC;
 2. The angles on the left of EF, viz. the exterior angle EGA, with its interior and opposite on the same side, GHC; and
 3. The two interior angles AGH and GHC.

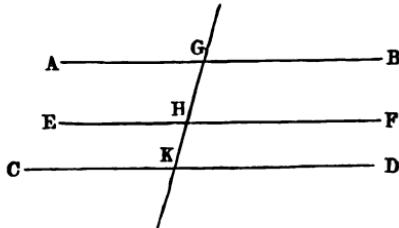
PROP. XXX. THEOREM.

Straight lines which are parallel to the same straight line are parallel to each other.

Let AB and CD be straight lines, parallel to the same straight line EF.

Then it is to be proved that

The straight lines AB and CD are parallel.



CONSTRUCTION.—Let the straight line GHK cut the straight lines AB, EF, and CD, in the points G, H, and K, respectively.

PROOF.—Because the straight lines AB and EF are parallel (hyp.), therefore the alternate angle AGH = the alternate angle GHF (I. 29).

Next, because the straight lines EF and CD are parallel (hyp.), therefore the exterior angle GHF = the interior and opposite angle HKD (I. 29).

But it has been just proved that the angle AGH = the angle GHF; therefore the angle AGH = the angle HKD, i.e. the angle AGK = the angle GKD (note 2 def. 15); and these are alternate angles.

Therefore, it is proved, as required, that

The straight lines AB and CD are parallel (I. 27).

Wherefore,

Straight lines, &c.

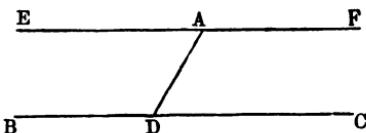
Q. E. D.

PROP. XXXI. PROBLEM.

To draw a straight line through a given point, parallel to a given straight line.

Let A be the given point, and BC the given straight line.

It is required to draw a straight line through A, which shall be parallel to BC.



CONSTRUCTION.—1. In the straight line BC, take any point, D, and join AD.

2. At the point A in the straight line AD make the angle EAD = the angle ADC (I. 23), and produce the straight line EA to F.

Then it is to be proved that

The straight line EF drawn through the point A, is parallel to the straight line BC.

PROOF.—*Because* the straight line AD falling upon the two straight lines EF and BC makes the alternate angle EAD = the alternate angle ADC (cons.), *therefore* EF is parallel to BC (I. 27).

Therefore, it is proved, as required, that

The straight line EF drawn through the point A is parallel to the straight line BC.

Q. E. F.

Exercise.

If from the extremities of two equal and parallel straight lines AB and CD, straight lines AD and BC are drawn, joining their extremities and intersecting in E; *prove* that AE = ED and that BE = EC.

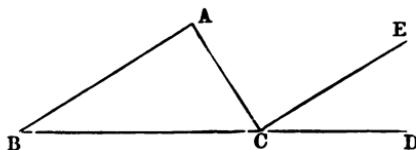
PROP. XXXII. THEOREM.

If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles of every triangle are together equal to two right angles.

Let ABC be a triangle, and let one of its sides, BC, be produced to D.

Then it is to be proved that

1. The exterior angle ACD = the two interior and opposite angles CAB and ABC; and
2. The three interior angles of the triangle, ABC, BCA, and CAB = two right angles.



CONSTRUCTION.—Through the point C draw CE parallel to BA (I. 31.)

PROOF.—1. Because AB is parallel to CE (cons.), and AC falls upon them, therefore the alternate angle ACE = the alternate angle BAC (I. 29).

Also because AB is parallel to CE (cons.), and BD falls upon them, therefore the exterior angle ECD = the interior and opposite angle ABC (I. 29).

We have therefore the angles ACE and ECD = the angles BAC and ABC. But the angles ACE and ECD make together the angle ACD, and therefore the angle ACD = the angles BAC and ABC (ax. 2).

Therefore, it is proved, as required, that

1. The exterior angle ACD = the two interior and opposite angles, CAB and ABC.
2. Next. Because, as we have just proved, the angle ACD = the angles BAC and ABC, add to each of these equals the

ngle ACD, and then the angles ACD and ACB = the angles CAB, ABC, and ACB (ax. 2).

But the angles ACD and ACB = two right angles I. 13), and therefore also the angles CAB, ABC, and BCA = two right angles (ax. 1).

Therefore, it is proved, as required, that

2. The three interior angles of the triangle, ABC, BCA, and CAB = two right angles.

Wherefore,

If a side of any triangle, &c.

Q. E. D.

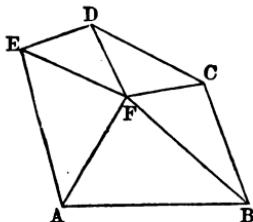
Exercises.

1. If AD bisecting the vertical angle A of a triangle ABC, bisects also the base BC in D; prove that ABC is an isosceles triangle.
2. If ABC be a triangle right-angled at A, and AD be drawn to bisect the base BC in D; prove that AD = BD or DC.
3. If ABC be an isosceles triangle with vertex A, and the side BA be produced beyond A to D; prove that the exterior angle CAD = twice the angle ABC or the angle ACB, the angles at the base.

COROLLARIES.

Corollary I

All the interior angles of any rectilineal figure, together with } = { Twice as many right angles as the figure has sides.
 four right angles }



Because any rectilineal figure, as ABCDE, can, by drawing straight lines from a point F, within the figure, to each angle, be divided into *as many triangles as the figure has sides*, as is shewn in the above figure, *and because* the three interior angles of every triangle equal two right angles (I. 32),

Therefore

All the interior angles of the triangles } = { Twice as many right angles as there are triangles.
 = { Twice as many right angles as the figure has sides.

But because

All the interior angles of the triangles } = { The interior angles of the figure with the angles at F;

And because

The angles at F = four right angles (I. 15, Cor. 2);

Therefore

All the interior angles of the figure with four right angles } = { All the interior angles of the triangles;
 = { Twice as many right angles as the figure has sides.

Similarly it may be demonstrated for any other rectilineal figure.

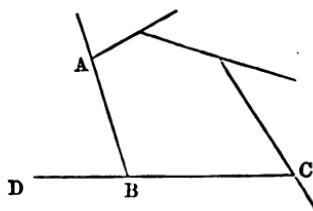
Therefore, it is proved, as required, that

All the interior, &c.

Q. E. D.

Corollary II.

All the exterior angles of any
rectilineal figure together } = { Four right angles.



Because the interior angle ABC with its adjacent exterior angle ABD = two right angles (I. 13),

Therefore

All the interior with all the ex- } = { Twice as many right
terior angles of the figure } angles as the figure
 has sides.

But because

All the interior angles of the } = { Twice as many right
figure with four right angles } angles as the figure has
 sides (I. 32, Cor. 1),

Therefore

All the interior with all the exterior angles of } = { All the interior angles of
the figure of } the figure with four right
 angles (ax. 1).

And taking away from each the *interior* angles common to both,

Then

All the exterior angles of the figure = four right angles
(ax. 3).

Similarly it may be demonstrated for any other rectilineal figure.

Therefore, it is proved, as required, that

All the exterior angles of any } = Four right angles.
rectilineal figure }

Q. E. D.

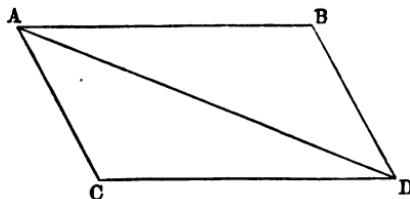
PROP. XXXIII. THEOREM.

The straight lines which join the extremities of two equal and parallel straight lines, towards the same parts, are also themselves equal and parallel.

Let AB and CD be two equal and parallel straight lines, joined towards the same parts by the straight lines AC and BD.

Then it is to be proved that

1. The straight lines AC and BD = each other.
2. The straight lines AC and BD are parallel to each other.



CONSTRUCTION.—Join AD (post. 1).

PROOF.—1. Because AB is parallel to CD, and AD meets them (hyp.), therefore the alternate angle $BAD =$ the alternate angle ADC (I. 29).

Next, because in the triangles BAD and ADC, we have the sides BA and AD, and their angle BAD , in the former = the sides CD and DA, and their angle CDA , in the latter, each to each (hyp. cons. and proof above), therefore the angle $ADB =$ the angle DAC , and the base $AC =$ the base BD (I. 4).

Therefore, it is proved, as required, that

1. The straight lines AC and BD = each other.

2. Further, because the straight line AD meets the two straight lines AC and BD, and makes the alt. angle ADB = the alt. angle DAC, as above demonstrated,

Therefore, it is proved, as required, that

2. The straight lines AC and BD are parallel to each other.

Wherefore,

The straight lines, &c.

Q. E. D.

Exercises.

1. Prove the above Proposition by joining BC.
2. Prove Prop. XXXII. by producing the side CB beyond B to D.
3. If ABC be a triangle with vertex A and base BC; prove that the line DF bisecting AB and AC in D and F respectively is parallel to the base BC.

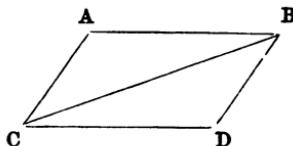
PROP. XXXIV. THEOREM.

The opposite sides and angles of a parallelogram are equal to one another, and the diameter bisects the parallelogram, that is, divides it into two equal parts.

Let ABDC be a parallelogram, of which BC is a diameter.

Then it is to be proved that

1. The opposite sides and angles of the parallelogram are equal to one another, viz., $AB = CD$, and $AC = BD$, the angle $CAB =$ the angle CDB , and the angle $ABD =$ the angle ACD ; and
2. The diameter BC divides the parallelogram into two equal parts.



PROOF.—1. Because AB is parallel to CD (def. 34) and BC meets them, therefore the alternate angle $ABC =$ the alternate angle BCD (I. 29).

And because AC is parallel to BD (hyp.) and BC meets them, therefore the alternate angle $ACB =$ the alternate angle CBD (I. 29).

Next, because in the triangles ABC and BCD we have the angles ABC and BCA , and the adjacent side BC in the former = the angles BCD and CBD and the adjacent side BC in the latter, each to each (cons. and I. 29), therefore the side AB = the side CD , and the side AC = the side BD , and the angle CAB = the angle CDB (I. 26).

Also, because the angles ABC and ACB = the angles BCD and CBD , each to each, as here proved, i.e. the whole angle ABD = the whole angle ACD ,

Therefore, it is proved, as required, that

1. The opposite sides and angles of a parallelogram are equal to one another.
2. Next, because in the triangles ABC and BCD we have the sides AB and BC, and their angle ABC, in the former = the sides DC and CB, and their angle DCB, in the latter, each to each, as here proved ; therefore the triangle ABC = the triangle BCD (I. 4).

Therefore, it is proved, as required, that

2. The diameter divides the parallelogram into two equal parts.

Wherefore,

The opposite sides, &c.

Q. E. D.

Exercises.

1. Prove Prop. XXXIV. by joining AD.
2. If ABC be an isosceles triangle with the vertical angle A a right angle ; prove that each of the angles at the base, ABC and ACB = half a right angle.
3. If ABCD be a parallelogram on base AB, and AC and BD its diagonals, intersecting in E ; prove that AE and BE = CE and DE, each to each.

PROP. XXXV. THEOREM.

Parallelograms on the same base, and between the same parallels, are equal to each other.

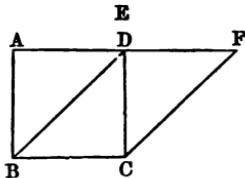
Let ABCD and EBCF be parallelograms on the same base BC, and between the same parallels AF and BC.

Then it is to be proved that

The parallelogram ABCD = the parallelogram EBCF.

This Proposition is considered under three Cases.

CASE I.—In this Case the sides AD and EF opposite to the base BC are *both terminated in point D*.

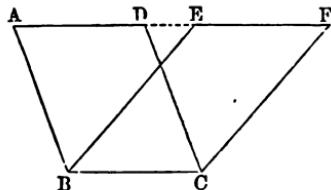


PROOF.—Because the triangle BDC is half the parallelogram ABCD (I. 34), and because the triangle BDC is also half the parallelogram EBCF (I. 34),

Therefore, it is proved in this Case, as required, that
(ax. 6)

The parallelogram ABCD = the parallelogram EBCF.

CASE 2.—In this Case let there be a space *between* the sides AD and EF opposite to the base BC.



PROOF.—Because ABCD is a parallelogram (hyp.), there-

fore $\overline{AD} = \overline{BC}$ (I. 34), and for the same reason $\overline{EF} = \overline{BC}$, therefore $\overline{AD} = \overline{EF}$ (ax. 1); then add \overline{DE} to each, and the whole $\overline{EA} =$ the whole \overline{FD} (ax. 2); also, $\angle A = \angle D$ (I. 34), and the exterior angle $\angle FDC =$ the interior and opposite angle $\angle EAB$ (I. 29).

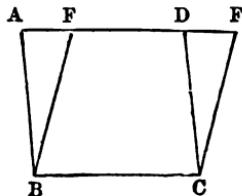
Now, because in the triangles EAB and FDC we have the sides \overline{EA} and \overline{AB} and their angle $\angle EAB$, in the former = the sides \overline{FD} and \overline{DC} and their angle $\angle FDC$, in the latter, each to each (as just proved), therefore the triangle EAB = the triangle FDC (I. 4).

Next, if we take the triangle EAB from the trapezium $FABC$, we have left the parallelogram $EBCF$; and if we take the triangle FDC from the same trapezium $FABC$, we have left the parallelogram $DABC$; and these remainders are equal (ax. 3).

Therefore, it is proved in this Case, as required, that

The parallelogram $ABCD$ = the parallelogram $EBCF$.

CASE 3.—In this case the sides \overline{AD} and \overline{EF} opposite the base \overline{BC} overlap each other as in the following figure.



PROOF.—The same method of proof applies in this Case as in Case 2, the only difference being that in Case 2 we added \overline{ED} to \overline{AD} and \overline{EF} , and here we have to take \overline{ED} away from each.

Wherefore,

Parallelograms on the same base, &c.

Q. E. D.

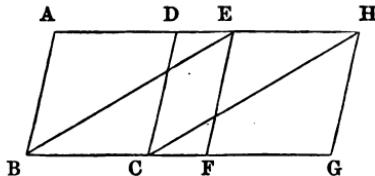
PROP. XXXVI. THEOREM.

Parallelograms on equal bases and between the same parallels are equal to one another.

Let ABCD and EFGH be parallelograms on equal bases ~~BC~~ and FG, and between the same parallels AH and BG.

Then it is to be proved that

The parallelogram ABCD = the parallelogram EFGH



CONSTRUCTION.—Join BE and CH.

PROOF.—Because $BC = FG$ (hyp.), and because $FG = EH$ (I. 34), therefore $BC = EH$ (ax. 1); also, because BC and EH are parallels and joined towards the same parts by BE and CH (hyp.), therefore BE and CH are both equal and parallel (I. 33); and therefore the figure EBCH is a parallelogram (def. 34).

Next, because the parallelograms ABCD and EBCH are upon the same base BC and between the same parallels AH and BC, therefore the parallelogram ABCD = the parallelogram EBCH (I. 35).

Similarly, the parallelogram EFGH = the parallelogram EBCH, being upon the same base EH, and between the same parallels EH and BG.

Therefore, it is proved, as required, that (ax. 1)

The parallelogram ABCD = the parallelogram EFGH.

Wherefore,

Parallelograms on equal bases, &c.

Q. E. D.

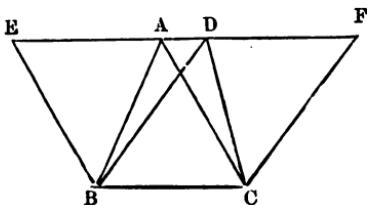
PROP. XXXVII. THEOREM.

Triangles on the same base and between the same parallels are equal to one another.

Let ABC and DBC be triangles on the same base BC, and between the same parallels AD and BC.

Then it is to be proved that

The triangle ABC = the triangle DBC.



CONSTRUCTION.—1. Produce AD both ways to points E and F.

2. Through B draw BE parallel to CA, and through C draw CF parallel to BD (I. 31).

PROOF.—*Because* each of the figures EBCA and DBCF is a parallelogram (def. 34), and because they are on the same base BC, and between the same parallels BC and EF, therefore the parallelogram EBCA = the parallelogram DBCF (I. 35).

Next, because the triangle ABC is half the parallelogram EBCA, and the triangle DBC is half the parallelogram DBCF (I. 34),

Therefore, it is proved, as required, that

The triangle ABC = the triangle DBC (ax. 7).

Wherefore,

Triangles on the same base, &c.

Q. E. D.

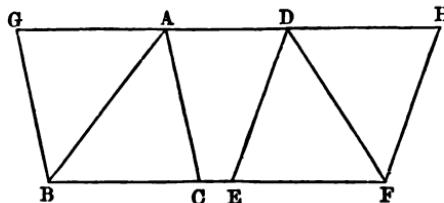
PROP. XXXVIII. THEOREM.

Triangles on equal bases and between the same parallels are equal to one another.

Let ABC and DEF be triangles on equal bases BC and EF, and between the same parallels AD and BF.

Then it is to be proved that

The triangle ABC = the triangle DEF.



CONSTRUCTION.—1. Produce AD both ways to G and H.

2. Through B draw BG parallel to CA, and through F draw FH parallel to ED (I. 31).

PROOF.—Because each of the figures GBCA and DEFH is a parallelogram (def. 34), and because they are on equal bases BC and EF, and between the same parallels GH and BF, therefore the parallelogram GBCA = the parallelogram DEFH (I. 36).

Next, because the triangle ABC is half the parallelogram GBCA, and because the triangle DEF is half the parallelogram DEFH (I. 34),

Therefore, it is proved, as required, that (ax. 7)

The triangle ABC = the triangle DEF.

Wherefore,

Triangles on equal bases, &c.

Q. E. D.

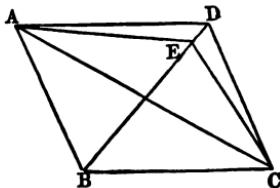
PROP. XXXIX. THEOREM.

Equal triangles on the same base, and on the same side of it are between the same parallels.

Let ABC and DBC be equal triangles, on the same base BC and on the same (viz. the upper) side of it.

Then it is to be proved that

The triangles ABC and DBC are between the same parallels.



CONSTRUCTION.—1. Draw AD joining the vertices of the triangles (post. 1); then, if AD is not parallel to their base BC,
2. Through A draw AE parallel to this base BC, meeting BD in E (I. 31), and join EC.

PROOF.—*Because* the triangles ABC and EBC are on the same base BC (hyp.), and between the same supposed parallels AE and BC, therefore the triangle ABC = the triangle EBC (I. 37).

But the triangle ABC = the triangle DBC (hyp.), therefore, also, the triangle DBC = the triangle EBC (ax. 1), i.e. the whole = its part, which is impossible (ax. 9).

Therefore the supposition that AE is parallel to BC is erroneous; and similarly it can be proved that only AD, joining the vertices of the triangles, is parallel to their base BC.

Therefore, it is proved, as required, that

The triangles ABC and DBC are between the same parallels.

Wherefore,

Equal triangles on the same base, &c.

Q. E. D.

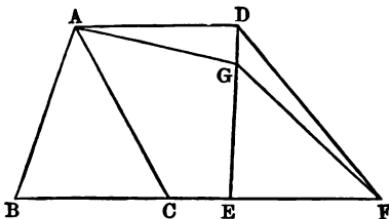
PROP. XL. THEOREM.

Equal triangles, on equal bases, in the same straight line, and on the same side of it, are between the same parallels.

Let ABC and DEF be equal triangles, on equal bases BC and EF, in the same straight line BF, and on the same (viz. the upper) side of it.

Then it is to be proved that

The triangles ABC and DEF are between the same parallels.



CONSTRUCTION.—1. Draw AD, joining the vertices of the triangles (post. 1); then, if AD is not parallel to the line of their bases BF,

2. Through A draw AG parallel to this line BF, meeting ED in G (I. 31), and join GF.

PROOF.—Because the triangles ABC and GEF are upon equal bases BC and EF (hyp.), and between the same supposed parallels AG and BF, therefore the triangle ABC = the triangle GEF (I. 38).

But the triangle ABC = the triangle DEF (hyp.), therefore, also, the triangle DEF = the triangle GEF (ax. 1), i.e. the whole = its part, which is impossible (ax. 9).

Therefore the supposition that AG is parallel to BF is erroneous; and similarly it can be proved that only AD, joining the vertices of the triangles, is parallel to the line of their bases, BF.

Therefore, it is proved, as required, that

The triangles ABC and DEF are between the same parallels.

Wherefore,

Equal triangles, on equal bases, &c.

Q. E. D.

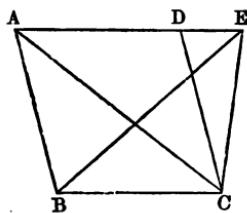
PROP. XLI. THEOREM.

If a parallelogram and a triangle be on the same base, and between the same parallels, the parallelogram shall be double of the triangle.

Let ABCD be a parallelogram, and EBC a triangle, on the same base BC, and between the same parallels BC and AE.

Then it is to be proved that

The parallelogram ABCD is double the triangle EBC.



CONSTRUCTION.—Join AC.

PROOF.—*Because* the triangles ABC and EBC are upon the same base BC, and between the same parallels BC and AE, therefore the triangle ABC = the triangle EBC (I. 37).

But the parallelogram ABCD is double the triangle ABC (L. 34).

Therefore, it is proved, as required, that

The parallelogram ABCD is double the triangle EBC.

Wherefore,

If a parallelogram and a triangle, &c.

Q. E. D.

Exercise.

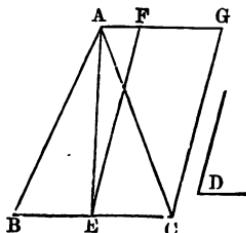
If ABCD be a quadrilateral with base AB and diagonals AC and BD, bisecting each other in E; prove that ABCD is a parallelogram.

PROP. XLII. PROBLEM.

To describe a parallelogram that shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle.

Let ABC be the given triangle, and D the given rectilineal angle.

It is required to describe a parallelogram = the triangle ABC, and having one of its angles = the angle D.



CONSTRUCTION.—1. Bisect BC in E (I. 10), and join AE.
2. At the point E in the line EC, make the angle CEF = the angle D (I. 23).

3. Through A draw AG parallel to EC, and through C draw CG parallel to EF (I. 31).

Then it is to be proved that

FECG is a parallelogram = the triangle ABC, and having the angle CEF = the angle D.

PROOF.—Because the triangles ABE and AEC are upon equal bases BE and EC (cons.), and between the same parallels BC and AG (cons.), therefore the triangle ABE = the triangle AEC (I. 38), and therefore the triangle ABC is double the triangle AEC.

But because the parallelogram FECG (cons. and def. 34) and the triangle AEC are upon the same base EC, and between the same parallels EC and AG, therefore the parallelogram FECG is double the triangle AEC (I. 41).

But we have seen that the triangle ABC is double the triangle AEC, therefore the parallelogram FECG = the triangle ABC (ax. 6), and it has one of its angles CEF = the angle D (cons.).

Therefore, it is proved, as required, that

FECG is a parallelogram = the triangle ABC, and having the angle CEF = the angle D.

Q. E. F.

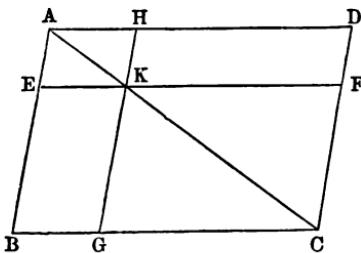
PROP. XLIII. THEOREM.

The complements of the parallelograms which are about the diameter of any parallelogram are equal to one another.

Let $ABCD$ be a parallelogram, of which AC is a diameter, with the parallelograms $AEKH$ and $KGCF$ about the diameter, i.e. through which the diameter AC passes.

Then the parallelograms $EBGK$ and $HKFD$, which make up the whole parallelogram $ABCD$, are the complements, and it is to be proved that

The complement $EBGK$ = the complement $HKFD$.



PROOF.—Because $ABCD$ is a parallelogram, and AC its diameter, therefore the triangle ABC = the triangle ADC (L. 34).

Similarly the triangle AEK = the triangle AHK , and the triangle KGC = the triangle KFC .

Hence the triangles AEK and KGC taken together = the triangles AHK and KFC taken together (ax. 2).

But we have seen that the whole triangle ABC = the whole triangle ADC ; therefore the remainder, or the parallelogram $EBGK$ = the remainder, or the parallelogram $HKFD$ (ax. 3); and these are the complements in the parallelogram $ABCD$.

Therefore, it is proved, as required, that

The complement $EBGK$ = the complement $HKFD$.

Wherefore,

The complements of the parallelograms, &c.

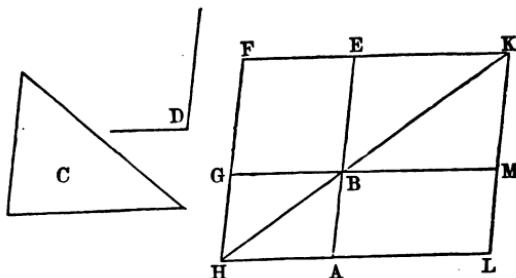
Q. E. D.

PROP. XLIV. PROBLEM.

To a given straight line to apply a parallelogram which shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle.

Let AB be the given straight line; C the given triangle; and D the given rectilineal angle.

It is required to apply to the straight line AB a parallelogram = the triangle C, and containing an angle = the angle D.



CONSTRUCTION (1).—1. Make the parallelogram BEFG = the triangle C, and with an angle EBG = the angle D (I. 42), so that BE may be in the same right line with AB.

2. Produce FG to H.

3. Through A draw AH parallel to BG or EF (I. 31), and join HB.

PROOF (1).—Because the straight line HF falls on the parallel lines AH and EF, therefore the two interior angles AHF and HFE = two right angles (I. 29), and therefore the angles BHF and HFE are less than two right angles; consequently the straight lines HB and FE will meet, if produced far enough (ax. 12).

CONSTRUCTION (2).—1. Produce the straight lines HB and FE to meet as in K.

2. Through K draw KL parallel to EA or FH (I. 31), and produce HA and GB to the points L and M.

Then it is to be proved that

ABML is a parallelogram = the triangle C, applied to the straight line AB, and having the angle ABM = the angle D.

PROOF (2).—*Because* FL* is a parallelogram (cons. and lef. 34) of which HK is a diameter, and FB and BL complements of the parallelograms GA and EM, about the diameter, *herefore* the parallelogram FB = the parallelogram BL (I. 43). *But* the parallelogram FB = the triangle C (cons.), *herefore* the parallelogram BL = the triangle C (ax. 1), and it is applied to the line AB, for it is one of its sides.

Again, *because* the angle GBE = the angle D (cons.), and *because* the angle ABM = the angle GBE (I. 15), *therefore* the angle ABM = the angle D (ax. 1).

Therefore, it is proved, as required, that

ABML is a parallelogram = the triangle C, applied to the straight line AB, and having the angle ABM = the angle D.

Q. E. F.

* N.B.—Parallelograms may be referred to by the two letters standing at *opposite* angles, instead of the four, at each angle. Thus the parallelogram FHLK may be referred to as the parallelogram FL, or KH.

Exercises.

1. If ABCD be a parallelogram with diagonal AC = diagonal BD; *prove* that each angle of the parallelogram A, B, C, and D is a right angle.

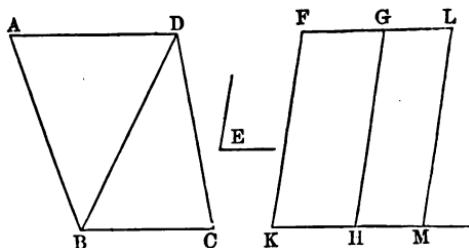
2. If ABCD be a parallelogram with its diagonals AC and BD intersecting at right angles in E; *prove* that the sides of the parallelogram AB, BC, CD, and DA = each other.

PROP. XLV. PROBLEM.

To describe a parallelogram equal to a given rectilineal figure, and having an angle equal to a given rectilineal angle.

Let ABCD be the given rectilineal figure, and E the given rectilineal angle.

It is required to describe a parallelogram = the figure ABCD, and having an angle = the angle E.



CONSTRUCTION.—1. Join BD and describe the parallelogram FH = the triangle ABD, and having the angle FKH = the angle E (I. 42).

2. Apply to the straight line GH the parallelogram GM equal to the triangle DBC, and having the angle GHM = the angle E (I. 44).

Then it is to be proved that

FM is a parallelogram = the figure ABCD, and having the angle FKM = the angle E.

PROOF.—*Because* each of the angles FKH and GHM = the angle E (cons.), therefore the angle FKH = the angle GHM (ax. 1); add to each the angle GHK, then the angles FKH and GHK = the angles GHM and GHK (ax. 2).

But the angles FKH and GHK = two right angles (I. 29); therefore the angles GHM and GHK = two right

ngles, and therefore KH is in the same straight line with IM (I. 14).

2. Next, because the lines FG and KM are parallel (cons.), and GH meets them, therefore the alternate angle MHG = the alternate angle GHM (I. 29); add to each the angle LGH, then the angles FGH and LGH = the angles GHM and LGH (ax. 2).

But the angles MHG and LGH = two right angles (I. 9); therefore the angles HGF and LGH = two right angles, and therefore FG is in the same right line with GL (I. 14).

3. Next, because KF is parallel to HG, and HG to ML (cons.), therefore KF is parallel to ML (I. 30), and KM and FL are also parallel (cons.); therefore the figure FKML is a parallelogram (def. 34).

Again, because the parallelogram FH = the triangle ABD (cons.), and the parallelogram GM = the triangle DBC (cons.), therefore the parallelogram FM = the figure ABCD, and it has the angle FKM = the angle E (cons.)

Therefore, it is proved, as required, that

FM is a parallelogram = the figure ABCD, and having an angle FKM = the angle E.

Q. E. F.

Corollary.—A parallelogram may be described equal to a given rectilineal figure, of any number of sides, and having one of its angles equal to a given angle.

N.B.—The learning of this Proposition will be simplified if it be observed—

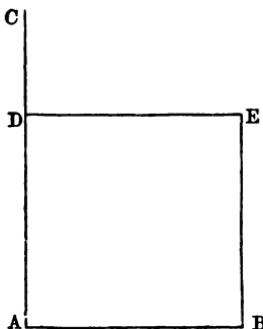
1. That the given figure is divided into triangles.
2. That a parallelogram is made equal to each triangle, the first according to Euclid I. 42, the second, and more if there be any, according to I. 44.
3. That the first part of the proof is to shew that KH and HM are in one and the same straight line.
4. That the second part is to shew that FG and GL are in one and the same straight line.

PROP. XLVI. PROBLEM.

To describe a square upon a given straight line.

Let AB be the given straight line.

It is required to describe a square upon AB .



CONSTRUCTION.—1. From A draw AC at right angles to and greater than AB (I. 11), cutting off $AD = AB$ (I. 3).

2. Through D draw DE parallel to AB : and through B draw BE parallel to AD (I. 31).

Then it is to be proved that

The figure $ABED$ is a square upon AB .

PROOF.—1. The figure $ABED$ is equilateral.

Because DE is parallel to AB (cons.), and BE is parallel to AD (cons.), therefore the figure $ABED$ is a parallelogram (def. 34), and therefore, also, $AB = DE$ and $AD = BE$ (I. 34).

But also $AB = AD$ (cons.), therefore the four sides AB , BE , ED , and DA = each other, and therefore the figure $ABED$ is equilateral, which was first to be shewn.

2. The figure $ABED$ is rectangular.

Next, because the lines AB and DE are parallel (cons.), and the line AD falls upon them, therefore the two interior angles BAD and ADE = two right angles (I. 29).

But because BAD is a right angle (cons.), therefore ADE a right angle (ax. 3); and because the opposite angles of parallelograms are equal to each other (I. 34), therefore the angles BED and ABE are also right angles, and therefore each of the angles of the figure = a right angle, and the figure ABED is rectangular. Which was next to be shown.

Hence, the figure ABED, being equilateral and rectangular is a square (def. 30), and it is described upon AB (cons.).

Therefore, it is proved, as required, that

The figure ABED is a square upon AB.

Q. E. F.

Exercises.

1. On a given straight line MN construct a square, with perp. NP drawn from N.
2. About MO, as a diagonal, construct a square MNOP.

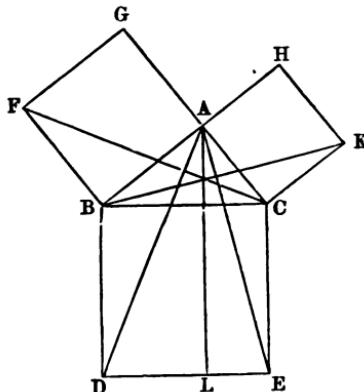
PROP. XLVII. THEOREM.

In any right-angled triangle, the square which is described upon the side subtending the right angle equals the sum of the squares described upon the sides containing the right angle.

Let ABC be a right-angled triangle having the right angle BAC.

Then it is to be proved that

The square described upon BC, the side *subtending*—i.e. *opposite to*—the right angle BAC = the sum of the squares described upon BA and AC, the sides *containing* that angle.



CONSTRUCTION.—1. On BC describe the square BDEC: on BA describe the square BAGF: and on AC describe the square AHKC (I. 46).

2. Through A draw AL parallel to BD or CE (I. 31), and join AD and FC.

PROOF.—*Because* the adjacent angles BAC and BAG = two right angles (hyp. and cons.), therefore CAG is one and the same straight line (I. 14).

And, for the same reason, BAH is one and the same straight line.

Next, because the angles DBC and FBA are each a right angle (cons.), therefore the angle DBC = the angle FBA; add to each the angle ABC, and therefore the angle DBA = the angle FBC (ax. 2).

And because in the triangles ABD and FBC we have the sides AB and BD and their angle ABD in the former = the sides FB and BC and their angle FBC in the latter, each to each, therefore the triangle ABD = the triangle FBC (I. 4).

Again, because the parallelogram BL and the triangle ABD are upon the same base BD, and between the same parallels AL and BD, therefore the parallelogram BL is double the triangle ABD (I. 41). And, for the same reason, the square GB is double the triangle FBC. But, as we have proved, the triangle ABD = the triangle FBC; therefore the parallelogram BL = the square GB (ax. 6).

In the same manner, by joining AE and BK, it is proved that the parallelogram CL = the square HC, and therefore the whole square BDEC = the sum of the squares GB and HC (ax. 2). But BDEC is the square on BC, subtending the right angle BAC, and GB and HC are the squares on BA and AC, the sides containing the right angle BAC;

Therefore, it is proved, as required, that

The square described on the side BC subtending the right angle = the sum of the squares described on the sides BA and AC, containing that angle.

Wherefore,

In any right-angled triangle, &c.

Q. E. D.

Exercises.

1. Prove, as stated in the above proposition, that the parallelogram CL = the square HC.

2. Prove that if in the figure of Prop. 47 the points E and K are joined by the line EK, and the points D and F by the line DF, the triangles KCE and FBD are equal to each other.

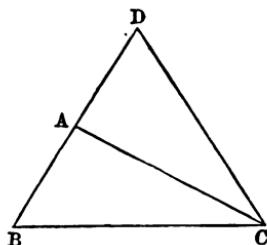
PROP. XLVIII. THEOREM.

If the square described upon one of the sides of a triangle be equal to the squares described upon the other two sides of it, the angle contained by these two sides is a right angle.

Let ABC be a triangle, with the square described upon BC = the sum of the squares described upon BA and AC.

Then it is to be proved that

The angle BAC is a right angle.



CONSTRUCTION.—From A draw AD at right angles to AC (I. 11), make AD equal to AB (I. 3), and join DC.

PROOF.—*Because* $AD = AB$ (cons.), *therefore* the square on $AD =$ the square on AB ; *and if* the square on AC be added to each of these equals, *then* the squares on AD and $AC =$ the squares on AB and AC (ax. 2).

And *because* the angle DAC is a right angle (cons.), *therefore* the square on $DC =$ the squares on AD and AC (I. 47).

Also the square on $BC =$ the squares on AB and AC (hyp.); *therefore* the square on $DC =$ the square on BC (ax. 1), and *therefore* $DC = BC$.

Next, *because* in the triangles BAC and DAC we have the sides BA , AC , and CB in the former = the sides DA ,

C, and CD in the latter, each to each (cons. and proof
above), therefore the angle BAC = the angle DAC (I. 8), and
cause the angle DAC is a right angle (cons.),

Therefore, it is proved, as required, that,

The angle BAC is a right angle.

Wherefore,

If the square, &c.

Q. E. D.

ADDENDUM.

BOOK I.

CLASSIFICATION OF PROPOSITIONS IN BOOK I.

I.

Straight Lines.

Props. 2, 3, 10, 11, 12, 14.

II.

Angles in connection with Straight Lines.

Props. 9, 13, 15, 23.

III.

Parallel Lines.

Props. 27, 28, 30, 31, 33.

IV.

Angles in connection with Parallel Lines.

Prop. 29.

V.

Construction of Triangles.

Props. 1, 7, 22.

VI.

Angles in connection with Triangles.

Props. 5, 16, 17, 18, 21, 32, 48.

VII.

Sides in connection with Triangles.

Props. 6, 19, 20, 47.

VIII.

Comparison of Triangles.

Props. 4 and 24, 8 and 25, 26.

IX.

Triangles and Parallel Lines.

Props. 37, 38, 39, 40.

X.

Triangles and Parallelograms.

Prop. 41

XI.

Properties and Comparison of Parallelograms.

Props. 34, 35, 36, 42, 43, 44, 45.

XII.

Construction of the Square.

Prop. 46.

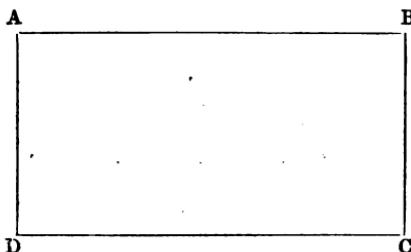
BOOK II.



DEFINITIONS.

I.

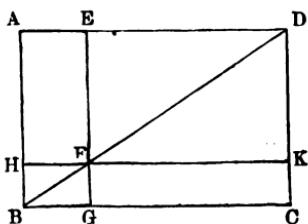
A Rectangle, *or* right-angled parallelogram, is said to be contained by any two of the straight lines which contain one of the right angles.



1. The rectangle ABCD is said to be contained by AB and BC, *or* by BC and CD, *or* by CD and DA, *or* by DA and AB.
2. The expression 'the rectangle AB, BC,' is allowed to be used instead of the larger one 'the rectangle contained by AB and BC.'
3. The rectangle is often referred to by the two letters standing at its opposite angles, as, in the above, rect. AC, *or* rect. DB, or by AC *or* BD only.

II.

very parallelogram the figure composed of either of parallelograms about the diameter, together with the two agents, is called a *gnomon*.



the parallelogram ABCD, the parallelogram EK about either, together with the complements AF and FC, forms the AKG.

similarly the parallelogram HG, about the diameter, together with the complements AF and FC, forms the *gnomon* EHC.

The *gnomon* is briefly expressed by the letters at the opposite corners of the parallelograms composing it. Thus the first gnomon AKG, composed of EK, AF, and FC, may be termed the AKG or HEC. Also the second gnomon EHC, composed of AF, and FC, may be termed the gnomon EHC or AGK.

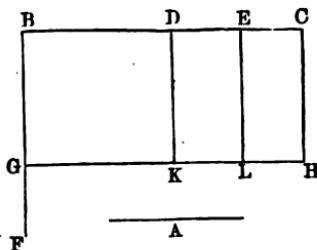
PROP. I. THEOREM.

If there be two straight lines, one of which is divided into any number of parts, the rectangle contained by the two straight lines is equal to the rectangles contained by the undivided line and the several parts of the divided line.

Let A and BC be two straight lines, one of which, BC, is divided into parts in D and E.

Then it is to be proved that

The rectangle contained by
the two straight lines A } = { The rectangles contained by
and BC } A and BD, by A and DE,
and by A and EC.



CONSTRUCTION.—1. From B draw BF at right angles to BC (I. 11), and cut off BG = A (I. 3).

2. Through G draw GH parallel to BC, meeting CH drawn parallel to BG (I. 31).

3. Through D and E draw DK and EL parallel to BG or CH (I. 31).

PROOF.—1. It is evident that the figure BH = the sum of the figures BK, DL, and EH.

2. But because BH is contained by BG and BC, of which BG = A (cons.), therefore BH is the rectangle contained by A and BC, the two given straight lines.

3. *Similarly* BK, DL, and EH are respectively the rectangles contained by BG and BD, by DK and DE, and by CL and EC—*i.e.* the rectangles contained by A and BD, by A and DE, and by A and EC (cons. and I. 34).

Therefore, it is proved, as required, that

The rectangle contained by the two straight lines A and BC = the rectangles contained by A and BD, by A and DE, and by A and EC.

Wherefore,

If there be two straight lines, &c.

Q. E. D.

Exercise.

Prove the above Proposition with MN and O, the given straight lines, MN being divided into parts in P, Q, and R.

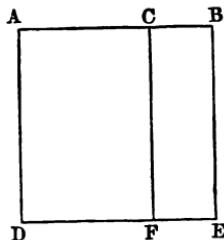
PROP. II. THEOREM.

If a straight line be divided into any two parts, the rectangles contained by the whole line and each of its parts are together equal to the square on the whole line.

Let AB be a straight line divided into any two parts in C.

Then it is to be proved that

The rectangles contained by AB and AC and by AB and BC together } = { The square on AB.



CONSTRUCTION.¹—1. On AB describe the square ADEB (I. 46).

2. Through C draw CF parallel to AD or BE (I. 31).

PROOF.—1. It is evident that the figure AE = the sum of the figures AF and CE.

2. But because AE is the square on AB (cons.), and because AF and CE are respectively the rectangles AD, AC, and CF, CB, i.e. the rectangles AB, AC, and AB, CB (cons., and I. 34),

Therefore, it is proved, as required, that

The rectangles contained by AB and AC and by AB and BC together = the square on AB.

Wherefore,

If a straight line be divided, &c.

Q. E. D.

¹ It may be taken as a rule in constructing the figures Prop. II.-VIII. inclusive, in this Book, that the first thing to be done is to construct the square spoken of in the Enunciation, and if more than one square is mentioned, the larger to be constructed.

Exercise.

If MN be a given straight line divided into any two parts in O, prove that

The rectangles contained by MN and MO, and by MN and NO, together = the square on MN.

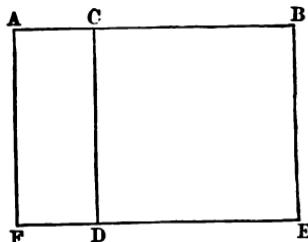
PROP. III. THEOREM.

If a straight line be divided into any two parts, the rectangle contained by the whole and one of the parts is equal to the rectangle contained by the two parts, together with the square on the aforesaid part.

Let AB be a straight line divided into any two parts in C .

Then it is to be proved that

The rectangle $\{ AB, BC \} = \{ \text{The rectangle } AC, CB, \text{ together with the square on } BC \}$.



CONSTRUCTION.—1. On BC describe the square $CDEB$ (I. 46).

2. Produce ED to F , meeting AF drawn parallel to CD or BE (I. 31).

PROOF.—1. It is evident that the figure $AE =$ the sum of the figures AD and CE .

2. But because AE is contained by AB and AF , of which $AF = CD$ (I. 34) $= BC$ (cons.), therefore AE is the rectangle AB, BC .

3. Next, because AD is the rectangle contained by AC , AF , of which $AF = BC$ (as before), therefore AD is the rectangle AC, CB .

4. Also CE is the square on BC (cons.).

Therefore, it is proved, as required, that

The rectangle AB, BC = the rectangle AC, CB, together with the square on BC.

Wherefore,

If a straight line be divided, &c.

Q. E. D.

Exercise.

Prove the other case of the above Proposition :

The rectangle AB, AC = the rectangle AC, CB, together with the square on AC.

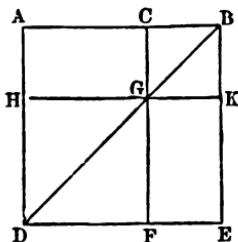
PROP. IV. THEOREM.

If a straight line be divided into any two parts, the square on the whole line is equal to the squares on the two parts, together with twice the rectangle contained by the parts.

Let AB be a straight line, divided into any two parts in C .

Then it is to be proved that

The square on $AB = \begin{cases} \text{The squares on } AC \text{ and } CB \text{ together} \\ \text{with twice the rectangle } AC, CB. \end{cases}$



CONSTRUCTION.¹—1. On AB describe the square $ADEB$ (I. 46) and join BD .

2. Through C draw CF parallel to AD or BE (I. 31) cutting BD in G .

3. Through G draw HGK parallel to AB or DE .

PROOF.²—1. Because CF and AD are parallel, and BD falls upon them (cons.), therefore ext. angle $BGC =$ int. opp. angle GDA (I. 29), i.e. the angle BDA (note def. 15).

Next, because $AB = AD$ (cons.), therefore the angle $ABD =$ the angle ADB (I. 5), i.e. the angle $CBG =$ the angle CGB (ax. 1), and therefore $CB = CG$ (I. 6); and since CB and $CG = GK$ and BK respectively (I. 34), therefore CB , CG , GK , and $BK =$ each other (ax. 1), and therefore the figure CK is equilateral.

2. Further, because CG and BK are parallels, and CB falls upon them (cons.), therefore the two int. angles GCB and $CBK =$ two right angles (I. 29). But because CBK is a right angle (cons.) therefore GCB is a right angle (ax. 1); and since the angles CBK and $GCB =$ the angles CGK and GKB respectively (I. 34), therefore the angles CBK , GCB , CGK ,

nd GKB each = a right angle, and therefore the figure CK is rectangular. It has also been proved to be equilateral, and therefore CK is a square (def. 30), and it is described on the side CB.

3. Similarly the figure HF is a square, on the side HG = AC (I. 34).

4. Next, because the comp. AG = the comp. GE (I. 43),
and also = the rect. AC, CG,
= the rect. AC, CB (cons.),
therefore the comp. GE = the rect. AC, CB (ax. 1);
and therefore the comps. AG and GE = twice the rect. AC, CB.

5. Now, it is evident that } = { the sum of the figures
the whole figure ADEB } = { AG, GE, HF, and CK.

But, as above, AG and GE = twice the rectangle AC, CB;
Also HF and CK = the squares on AC and CB respectively;
And the whole figure ADEB = the square on AB (cons.).

Therefore, it is proved, as required, that

The square on AB = the squares on AC and CB, together with twice the rectangle AC, CB.

Wherefore,

If a straight line be divided, &c.

Q. E. D.

Cor.—The parallelograms about the diameter of a square are squares.

¹ In addition to the note as to Construction, Prop. I., it may now be stated that the figures of Prop. IV.—VIII. require, after the square has been described, the drawing of the diagonal, and of the parallel, or parallels, employed.

² It will assist the learner to notice the following steps in this Proposition:—

- a. To prove that CK is the square on CB (as in 1 and 2).
- b. To observe that HF is the square on HG, i.e. on AC (as in 3).
- c. To prove that the complements AG and GE = twice rect. AC, CB (as in 4).
- d. To combine these conclusions is the proof of the Proposition itself (as in 5).

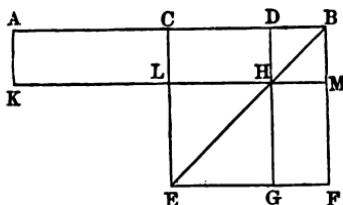
PROP. V. THEOREM.

If a straight line be divided into two equal parts, and also into two unequal parts, the rectangle contained by the unequal parts, together with the square on the line between the points of section, is equal to the square on half the line.

Let AB be a straight line divided into two equal parts in C, and into two unequal parts in D.

Then it is to be proved that

The rectangle AD, DB together
with the square on CD } = The square on CB.



CONSTRUCTION.—1. On CB describe the square CEFB (I. 46), and join BE.

2. Through D draw DHG parallel to CE or BF (I. 31), cutting BE in H.

3. Through H draw KLHM parallel to AB or EF, cutting CE in L and BF in M.

4. Through A draw AK parallel to CL or BM.

PROOF.—*Because* the comp. CH = the comp. HF (I. 43),

Therefore CH and DM = HF and DM (ax. 2),
i.e. the whole CM = the whole DF.

But because CM = AL (I. 36),
therefore AL = DF (ax. 1),
and therefore AL and CH = DF and CH (ax. 2.),
i.e. the rect. AH = the gnomon CMG.

But because AH is the rect. AD, DH, and DH = DB
 (I. 4, Cor.), therefore AH = the rect. AD, DB,

and therefore the gnomon CMG = the rect. AD, DB (ax. 1);
 therefore also the gnomon CMG together with LG, i.e. the
 whole figure CEFB = { the rect. AD, DB,
 together with LG;

but because LG is the square on LH (II.4, Cor.) and LH=CD
 (I. 34),

therefore the whole figure CEFB = { the rect. AD, DB, together
 with the square on CD.

But the whole figure CEFB = the square on CB (cons.),

Therefore, it is proved, as required, that

The rectangle AD, DB, together with the square on
 CD = the square on CB.

Wherefore,

If a straight line be divided, &c.

Q. E. D.

Exercises.

1. Prove that as stated in Prop. IV. Proof 3, the figure
IF is a square on the side HG.

2. Prove that in Prop. V. the rectangle AD, DB, together with the square on CD = the square on AC.

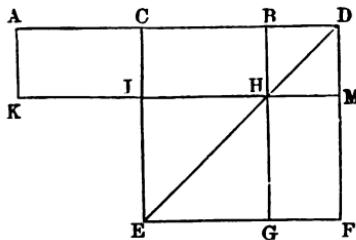
PROP. VI. THEOREM.

If a straight line be bisected, and produced to any point, the rectangle contained by the whole line thus produced, and the part of it produced, together with the square on half the line bisected, is equal to the square on the straight line which is made up of the half and the part produced.

Let AB be a straight line bisected in C, and produced to any point D.

Then it is to be proved that

The rectangle AD, DB, together } = The square on CD.
with the square on BC }



CONSTRUCTION.—1. On CD describe the square CEF^D (I. 46), and join DE.

2. Through B draw BG parallel to CE or DF (I. 31), cutting DE in H.

3. Through H draw KM parallel to AD or EF, cutting CE in L and DF in M.

4. Through A draw AK parallel to CL or DM.

PROOF.—

Because AL = CH (I. 36),

and CH = HF (I. 43),

therefore AL = HF (ax. 1);

*and therefore AL and CM = HF and CM,
i.e. AM = the gnomon CMG.*

But because AM is the rect. AD, DM , and $DM = DB$ [I. 4, Cor.], therefore $AM = \text{rect. } AD, DB$,
et therefore the gnomon CMG = the rect. AD, DB (ax. 1),
therefore also the gnomon CMG together with LG ,
i.e. the whole figure $CEFD$ = { the rect. AD, DB together with LG .

And because LG is the square on LH (II. 4, Cor.), and $H = CB$ (I. 34),
therefore the whole figure $CEFD$ = { the rect. AD, DB , together with the square on CB .

But the whole figure $CEFD$ = the square on CD (cons.).

Therefore, it is proved, as required, that

The rectangle AD, DB together with the square on BC
= the square on CD .

Wherefore,

If a straight line be bisected, &c.

Q. E. D.

Exercise.

Prove that if in the above Proposition we take AB as
sected in C , and produced *beyond A* to D , then we should
have,

The rectangle BD, AD together with the square on AC
= the square on CD .

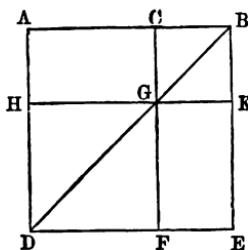
PROP. VII. THEOREM.

If a straight line be divided into any two parts, the squares on the whole line, and on one of the parts, are equal to twice the rectangle contained by the whole and that part, together with the square on the other part.

Let AB be a straight line divided into any two parts in C .

Then it is to be proved that

The squares on $AB\}$ = {Twice the rectangle AB, BC ,
and BC together with the square on AC .



CONSTRUCTION.—1. On AB describe the square $ADEB$ (I. 46), and join BD .

2. Through C draw CF parallel to AD or BE (I. 31), cutting BD in G .

3. Through G draw HK parallel to AB or DE .

PROOF.— *Because $AG = GE$ (I. 43),
therefore AG and $CK = GE$ and CK (ax. 2),
i.e. $AK = CE$;*

and therefore AK and $CE =$ twice AK .

*But AK and $CE =$ the gnomon AKF and CK ,
therefore the gnomon AKF and $CK =$ twice AK (ax. 1).*

Next, because AK is the rect. AB, BK, and BK = BC . 4, Cor.), therefore AK = the rect. AB, BC,

$$\{ \text{therefore the gnomon } AKF \} = \{ \text{twice the rect. AB, BC and CK} \} ; \quad (\text{ax. 1});$$

$$\{ \text{before also the gnomon } AKF \} = \{ \text{twice the rect. AB, BC with CK and HF} \} = \{ \text{and HF.}$$

But because HF is the square on HG, and HG = AC (I. 34),

$$\{ \text{before the gnomon } AKF \text{ with CK and HF} \} = \{ \text{twice the rect. AB, BC, together with the square on AC.}$$

$$\{ \text{with CK and HF together} \} = \{ \text{the whole figure ADEB with CK;}$$

$$\{ \text{id the whole figure ADEB together with CK} \} = \{ \text{the squares on AB and BC; (cons. and II. 4, Cor.)}$$

Therefore, it is proved, as required, that

The squares on AB and BC = twice the rectangle AB, BC, together with the square on AC.

Wherefore,

If a straight line be divided, &c.

Q. E. D.

Exercise.

Prove the other case in the above Proposition, that

The squares on AB and AC = twice the rectangle AB, together with the square on BC.

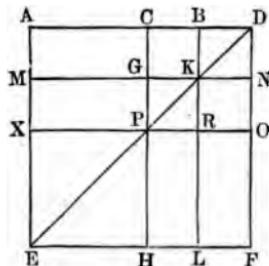
PROP. VIII. THEOREM.

If a straight line be divided into any two parts, four times the rectangle contained by the whole line and one of the parts, together with the square on the other part, is equal to the square on the straight line which is made up of the whole line and that part.

Let AB be a straight line divided into any two parts in C.

Then it is to be proved that

Four times the rectangle A.B, } together with the square } = { the square on AD,
BC, i.e. on AB and
on AC together.



CONSTRUCTION.—1. Produce AB to D, making $BD = BC$ (post. 2 and I. 3).

2. Upon AD describe the square AEDF (I. 46).

3. Complete the figure by the parallels cutting the diagonal, each way, in K and P (I. 31).

PROOF.—1. Because $BD = BC$ (cons.), and $= KN$ (I. 34), therefore BC and BD each $= KN$ (I. 34), therefore $GK = KN$ (ax. 1), and similarly $PR = RO$.

2. Next, because $CB = BD$ and $GK = KN$, therefore the rect. CK = the rect. BN, and the rect. GR = the rect. KO (I. 36).

3. But because the rect. CK = the rect. KO (I. 43), therefore the rect. BN = the rect. GR, and therefore the four rectangles CK, BN, GR, and KO = each other, therefore also they are, together, four times any one of them, as CK.

4. Next, because $BD = BC$ (cons.), and because $BD = BK$ (II. 4, Cor.) $= CG$ (I. 34), therefore $BC = CG$ (ax. 1); and because $BC = GK$ (I. 34) $= GP$ (II. 4, Cor.), therefore $CG = GP$ (ax. 1); and therefore the rect. AG = the rect. MP (I. 36);

l because PR = RO, therefore the rect. PL = the rect. RF 36). But because the rect. MP = the rect. PL (I. 43) before these four rectangles AG, MP, PL, and RF = each other, and therefore they are together four times any one of m, as AG.

5. And because the four rect-} = {four times the rect. CK, BN, GR, and KO} = {CK;

before the eight rectangles} = four times the rect. AK. making the gnomon AOH

d because AK is the rect. AB, BC, since BK = BD, before four times the rect. AB, BC = four times the rect. AK, = the gnomon AOH,

l therefore four times the rect.} = {the gnomon AOH AB, BC, with XH} = {with XH,

and because XH = the square on XP = AC,

before four times the rect. AB,} = {the gnomon AOH with BC, with the square on AC} = {the square on AC,

= the whole figure AEF D,

= the square on AD (cons.),

= {the square on AB and BC together.

Therefore, it is proved, as required, that

Four times the rectangle AB, BC, together with the square on AC = the square on AD, i.e. on AB and BC together.

Wherefore,

If a straight line be divided, &c.

Q. E. D.

N.B.—It will assist the learner to notice the following in this Proposition :—

a. To prove that the four rectangles CK, BN, GR, and KO = each other, and that together they are four times any one of them, as CK (as in 1, 2, 3).

b. To prove that the four rectangles AG, MP, PL, and RF = each other, and that together they are four times any one of them, as AG (as in 4).

c. To combine these conclusions is the proof of the Proposition itself (as in 5).

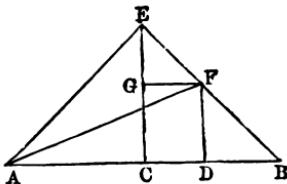
PROP. IX. THEOREM.

If a straight line be divided into two equal and also into two unequal parts, the squares on the two unequal parts are together double of the square on half the line and of the square on the line between the points of section.

Let AB be a straight line divided into two equal parts in C, and into two unequal parts in D.

Then it is to be proved that

$$\text{The squares on } AD \} = \left\{ \begin{array}{l} \text{double the squares on } AC \\ \text{and } DB \text{ together} \end{array} \right\} = \left\{ \begin{array}{l} \text{and } CD. \end{array} \right\}$$



CONSTRUCTION.—1. From C draw CE at right angles to AB and $= AC$ or CB (I. 11, and 3).

2. Join EA and EB.
3. Through D draw DF parallel to CE (I. 31), meeting EB in F.
4. Through F draw FG parallel to AB (I. 31), meeting CE in G, and join AF.

PROOF.—1. Because $AC = CE$ (cons.), therefore the angle $CEA =$ the angle CAE (I. 5); and because ACE is a right

angle (cons.), therefore the angles CEA and CAE are together a right angle (I. 32); and since they are equal to each other, therefore each angle CEA and CAE = half a right angle.

Similarly, each angle CEB and CBE = half a right angle, and therefore the angles AEC and CEB together, i.e. the angle AEB = a right angle.

2. Next, because in the triangle EGF the angle GEF = the angle CEB (note 2 def. 15) = half a right angle, and because the angle EGF is a right angle, since the angle EGF = the angle GCB (I. 29) = the angle ECB (note def. 15) = a right angle (cons.), therefore the remaining angle EFG = half a right angle (I. 32), and therefore the side EG = the side GF (I. 6).

3. Again, because in the triangle FDB the angle FBD = the angle EBC, as above = half a right angle; and because the angle FDB is a right angle (cons.); therefore the remaining angle BFD = half a right angle = the angle FBD, and therefore the side DF = the side DB (I. 6).

4. Because in the triangle AEC the side AC = the side CE (cons.), therefore the square on AC = the square on CE, and therefore the squares on AC and CE together = double the square on AC.

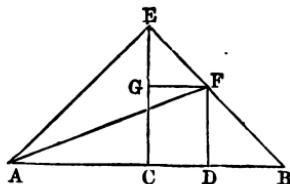
But because the square on AE = the squares on AC and CE (I. 47), therefore the square on AE = double the square on AC (ax. 1).

5. Because in the triangle GEF it has been proved that the side EG = the side GF, therefore the square on EG = the square on GF, and therefore the squares on EG and GF = double the square on GF.

But because the square on EF = the squares on EG and GF (I. 47), therefore the square on EF = double the square on GF.

And because GF = CD (I. 34), therefore the square on EF = double the square on CD (ax. 1).

And because, as already proved, the square on EA = double the square on AC, *therefore* the squares on EA and EF = double the squares on AC and CD (ax. 2).



6. Next, *because* in the triangle AEF the square on AF = the squares on EA and EF (I. 47), *therefore* the square on AF = double the squares on AC and CD (ax. 1).

7. But *because* in the triangle ADF the square on AF = the squares on AD and DF (I. 47), *therefore* the squares on AD and DF = double the squares on AC and CD (ax. 1); *and since*, as already proved, DF = DB,

Therefore, it is proved, as required, that

The squares on AD and DB together = double the squares on AC and CD.

Wherefore,

If a straight line be divided, &c.

Q. E. D.

N.B.—It will assist the learner to note the following steps in this Proposition :

- a. To prove that AEB is a right angle (as in 1).
- b. To prove that the side EG = the side GF (as in 2).
- c. To prove that the side DF = the side DB (as in 3).
- d. In the triangle AEC to prove that the square on AE = double the square on AC (as in 4).

- e. In the triangle GEF to prove that the square on EF
= double the square on CD (as in 5); and to prove
that the squares on AE and EF = double the squares
on AC and CD.
- f. In triangle AEF to prove that the square on AF
= double the squares on AC and CD (as in 6).
- g. In the triangle ADF to prove that the squares on
AD and DB = double the squares on AC and CD
(as in 7).

Exercises.

Prove the other case in Proposition VIII. that if
e produced to D, making $AD = AC$,
our times the rectangle contained by AB, AC, together
the square on BC = the square on BD, i.e. on AB and
ogether.

Prove the other case in Prop. IX. that if AB be
ed unequally between A and C,
he squares on AD and DB together = double the
es on BC and CD.

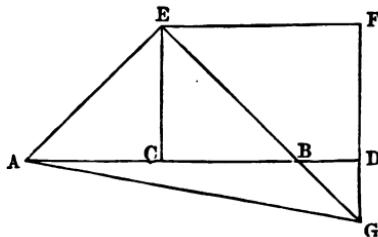
PROP. X. THEOREM.

If a straight line be bisected and produced to any point, the square on the whole line thus produced, and the square on the part of it produced, are together double of the square on half the line bisected and of the square on the line made up of the half and the part produced.

Let AB be a straight line bisected in C, and produced to D.

Then it is to be proved that

The squares on AD and DB = double the squares on AC and CD.



CONSTRUCTION (1).—1. Draw CE at right angles to AB (I. 11), and = AC or BC (I. 3).

2. Join AE and EB.

3. Draw EF parallel to CB, meeting DF drawn parallel to CE (I. 31).

Then

Because EF meets the parallels EC and FD, therefore the two int. angles CEF and EFD = two right angles (I. 29); and therefore the angles BEF and EFD are less than two right angles; and therefore EB

and FD will meet, if produced towards B and D (ax. 12).

CONSTRUCTION (2).—Let EB and FD be produced meeting in G, and join AG.

PROOF.—1. *Because* $AC = CE$ (cons.), therefore the angle $CEA =$ the angle CAE (I. 5); *and because* ACE is a right angle, *therefore* the angles CEA and CAE are together a right angle (I. 32); *and since* they are equal to each other, *therefore* each angle CEA and $CAE =$ half a right angle.

Similarly, each angle CEB and $CBE =$ half a right angle, *and therefore* the angles CEA and CEB together, viz. the angle AEB , i.e. the angle $AEG =$ a right angle.

2. *Because* in the triangle BDG the angle $DBG =$ the angle CBE (I. 15) = half a right angle, as just proved, *and because* the angle $BDG =$ the angle ECB (I. 29) = a right angle, *therefore* the remaining angle $DGB =$ half a right angle = the angle DBG , *and therefore* the side $BD =$ the side DG (I. 6).

3. *Because* in the triangle EGF , the angle $EGF =$ the angle $BGD =$ half a right angle, as just proved, *and because* the angle EFG or the angle $EFD =$ the angle ECD (I. 34) = a right angle, *therefore* the remaining angle $GEF =$ half a right angle = the angle EGF , *and therefore* the side $FE =$ the side FG (I. 6).

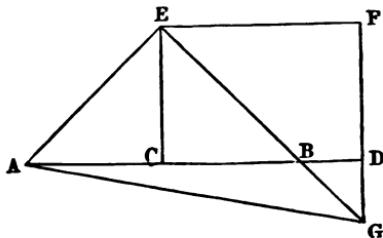
4. Again, *because* in the triangle EGF , the side $FE =$ the side FG , as just proved, *therefore* the square on $FG =$ the square on FE , *and therefore* the squares on FG and $FE =$ double the square on FE .

But *because* the square on $EG =$ the squares on FG and FE (I. 47), *therefore* the square on $EG =$ double the square on EF (ax. 1); *and since* $EF = CD$ (I. 34), *therefore* the square on $EG =$ double the square on CD .

5. Next, *because* in the triangle ACE , the side $AC =$ the side CE (cons.), *therefore* the square on $AC =$ the square on CE , *and therefore* the squares on AC and $CE =$ double the square on AC (ax. 1).

But *because* the square on $AE =$ the squares on AC and

CE (I. 47), therefore the square on AE = double the square on AC (ax. 1).



6. Now, because the square } = {double the square on
on AE

and because the square on EG = {double the square on
 CD , as above,

therefore the squares on AE and } = {double the squares on
 EG } = { AC and CD (ax. 2).

But the square on AG = {the squares on AE and
 EG (I. 47),

therefore the square on AG = {double the squares on
 AC and CD (ax. 1).

Again, the square on AG = {the squares on AD and
 DG (I. 47),

therefore, also, the squares on } = {double the squares on
 AD and DG } = { AC and CD ;

and since DG = DB , as already proved,

Therefore, it is proved, as required, that

The squares on AD and DB = double the squares on
 AC and CD .

Wherefore,

If a straight line be bisected, &c.

Q. E. D.

N.B.—It will assist the learner to note the following steps in this Proposition :—

- a. To prove that AEB , i.e. AEG , is a right angle (as in 1).
- b. To prove, in the triangle BDG , that $BD = DG$ (as in 2).
- c. To prove, in the triangle EGF , that $FE = FG$ (as in 3).
- d. To prove, also in the triangle EGF , that the square on $EG =$ double the square on EF , i.e. on CD (as in 4).
- e. To prove in the triangle ACE that the square on $AE =$ double the square on AC (as in 5).
- f. To prove from the triangles AEG and ADG , in connection with previous results, the assertion made in the Proposition.

Exercise.

Prove the other case in Prop. X. that if BA be produced to D , beyond A ,

The squares on BD and $AD =$ double the squares on BC and CD .

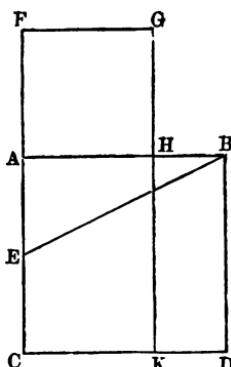
PROP. XI. PROBLEM.

To divide a straight line into two parts, so that the rectangle contained by the whole and one of the parts shall be equal to the square on the other part.

Let AB be a straight line.

It is required to divide AB into two parts, say in H, so that

The rectangle AB, BH = the square on AH.



CONSTRUCTION.—1. On AB describe the square ACDB (I. 46).

2. Bisect AC in E and join BE (I. 10, and post. 1).
3. Produce CA to F, making $EF = EB$ (post. 2, and I. 3).
4. On AF describe the square AFGH.
5. Produce GH to K, in CD.

Then it is to be proved that AB is divided in H, so that
The rectangle AB, BH = the square on AH.

PROOF.—The rect. CF, FA, } = { the square on EF
 with the square on EA } = { (II. 6).
 = { the square on EB, since
 EF = EB (cons.).

But because EAB is a right angle (cons.), therefore the } = { the squares on EA and square on EB } = { AB (I. 47),
 and therefore the rect. CF, FA, } = { the squares on EA and with the square on EA } = { AB ;
 therefore also the rect. CF, FA = the square on AB (ax. 3).

But FK is the rect. CF, FA, since FG = FA and AD is the square on AB (cons.),

therefore FK = AD.

Take away the common part } AK, then the remainder FH } = the remainder HD.

Now, FH is the square on AH (cons.), and HD is the rectangle DB, BH, i.e. AB, BH (cons.).

Therefore, it is proved, as required, that

The rectangle AB, BH = the square on AH ; and the straight line AB is divided into two parts in H, as required.

Q. E. F.

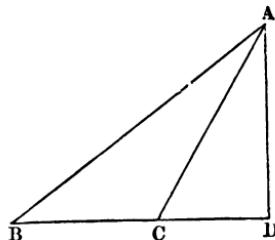
PROP. XII. THEOREM.

In obtuse-angled triangles, if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle, by twice the rectangle contained by the side on which, when produced, the perpendicular falls, and the straight line intercepted without the triangle, between the perpendicular and the obtuse angle.

Let ABC be an obtuse-angled triangle, having the obtuse angle ACB, and, from the acute angle A, let AD be drawn perpendicular to BC, produced to D.

Then it is to be proved that

The square on AB is *greater* than the squares on BC and CA by twice the rect. BC, CD.



PROOF.—*Because the square on BD = the squares on BC and CD, and twice the rect. BC, CD (II. 4), therefore the squares on BD and DA = the squares on BC, CD, and DA, and twice the rect. BC, CD (ax. 2).*

But because BDA is a right angle (hyp.), therefore the square on AB = the squares on BD and DA (I. 47).

Similarly, the square on CA = the squares on CD and DA.

Therefore, substituting these values (in line 2), the square on AB = the squares on BC and CA, with twice the rect. BC, CD.

Therefore, it is proved, as required, that

The square on AB is *greater* than the squares on BC
and CA by twice the rectangle BC, CD.

Wherefore,

In obtuse-angled triangles, &c.

Q. E. D.

Exercise.

Prove the other case in Prop. XII. that if the perpendicular BD be drawn from the other acute angle, B, to the side AC produced to D,

The square on AB is *greater* than the squares on BC and CA by twice the rectangle AC, CD.

PROP. XIII. THEOREM.

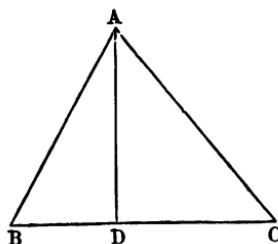
In every triangle the square on the side subtending an acute angle is less than the squares on the sides containing that angle, by twice the rectangle contained by either of these sides and the straight line intercepted between the perpendicular let fall on it from the opposite angle, and the acute angle.

Let ABC be any triangle, and the angle at B an acute angle; and on BC, one of the sides containing it, let fall the perpendicular AD from the opposite angle.

Then it is to be proved that

The square on AC is *less* than the squares on CB and BA by twice the rectangle CB, DB.

CASE I.—When the perpendicular falls *within* the triangle ABC.



PROOF.—*Because* the squares on CB and DB = twice the rect. CB, DB, with the square on DC (II. 7),

Therefore the squares on CB, DB, and DA = twice the rect. CB, DB, with the squares on DC and DA.

But because BDA is a right angle (hyp.),

Therefore the square on AB = the squares on DB and DA (I. 47).

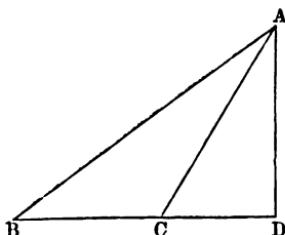
Similarly, the square on AC = the squares on AD and DC.

Therefore, substituting these values (in line 2), the squares on CB and AB = twice the rect. CB, DB with the square on AC.

Therefore, it is proved, as required in this Case, that

The square on AC is *less* than the squares on CB and BA by twice the rectangle CB, DE.

CASE II.—When the perpendicular falls *without the triangle ABC*.



PROOF.—*Because* the angle at D is a right angle (hyp.), and because the angle ACD is greater than the angle at D (I. 16), therefore the angle ACD is an *obtuse angle*;

and therefore the square on AB = {the squares on AC and CB, with twice the rect. CB, CD (II. 12);

therefore, also, the squares on } = {the squares on AC, CB, BC, with twice the rect. BC, CD,

= {the square on AC, twice the square on BC, with twice the rect. BC, CD.

But because the rect. BD, BC = {the rect. BC, CD, with
the square on BC (II.3),
therefore twice the rect. BD, BC = {twice the rect. BC, CD,
with twice the square
on BC;

therefore, also, substituting these values (in lines 4, &c.), the squares on AB and BC = the square on AC, with twice the rect. BD, BC.

Therefore, it is proved, as required in this Case, that

The square on AC is *less* than the squares on CB and BA by twice the rectangle CB, DB.

CASE III.—When the perpendicular is a *side of the triangle ABC*.



PROOF.—In this Case the side BC is the straight line between the perpendicular let fall from the opposite angle A, and the acute angle taken at B.

Now, because the square on AB = the squares on AC and CB (I. 47), therefore the squares on AB and BC = the square on AC, with twice the square on CB (ax. 2).

Therefore, it is proved, as required in this Case, that

The square on AC is *less* than the squares on CB and BA by twice the rectangle CB, CB, *i.e.* CB, CD, in other Cases.

Wherefore,

In every triangle the square, &c.

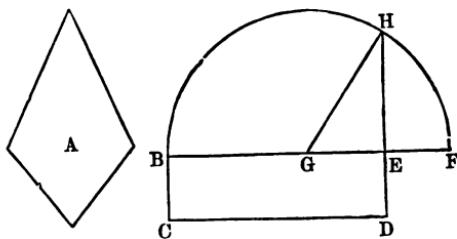
Q. E. D.

PROP. XIV. PROBLEM.

To describe a square that shall be equal to a given rectilineal figure.

Let A be the given rectilineal figure.

It is required to describe a square = the rectilineal figure A.



CONSTRUCTION.—1. Describe the rectangular parallelogram BCDE = A (I. 45), and if the sides BE and ED = each other, it is a square, and what was required is done.

2. But if BE and ED are not equal, then produce one of them as BE, to F, making EF = ED (post. 2, and I. 3).

3. Bisect BF in G (I. 10).

4. From centre G, with radius GB or GF, describe semi-circle BHF (post. 3).

5. Produce DE to H, and join GH (posts. 2 and 1).

Then it is to be proved that

The square described on EH = the rectilineal figure A.

PROOF.—*Because* the rect. BE, } = { the square on GF (II. EF, with the square on GE } = { 5,

= { the square on GH, for
GH = GF (cons.),

use the square on GH } = { the squares on GE and
the rect. BE, EF, with } = { the squares on GE and
square on GE } = { EH (ax. 1),
before the rect. BE, EF = the square on EH (ax. 3).
the rect. BE, EF, is the rect. BE, ED, for EF = ED ;
∴ the rect. BD = the square on EH.
the rect. BD = the figure A (cons.).

efore, it is proved, as required, that
the square described on EH = the rectilineal figure A.

Q. E. F.

A D D E N D U M.

EUCLID, BOOK II.

The chief difficulty the learner has in remembering, as well as in learning, the Propositions of Euclid, Book II., arises from the verbal similarity in many of the Enunciations.

This difficulty may be lessened, in preparing for an examination at least, by taking the following Propositions in the classes referred to.

A.

Properties of a straight line divided into any two parts.
Props. 2, 3, 4, 7, and 8.



Let AB be a straight line divided into any two parts in C.

Then, Prop. 2,

The rectangles contained by the whole and each of the parts } = the square on the whole line.

Prop. 3,

The rectangle contained by the whole and one part } = { the rectangle contained by two parts, with square on the aforesaid part.

Prop. 4,

The square on the whole line } = { the squares on the two parts with twice the rectangle contained by the parts.

Prop. 7,

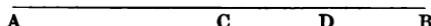
The squares on the whole line and one of the parts } = twice the rectangle contained by the whole and that part, together with the square on the other part.

Prop. 8,

Four times the rectangle contained by the whole and one part, together with the square on the other part, } = the square on the straight line made up of the whole and that part.

B.

Properties of a straight line divided equally and unequally.
Props. 5 and 9.



Let AB be a straight line divided equally in C and unequally in D.

Then, Prop. 5,

The rectangle contained by the unequal parts, together with the square on the line between the points of section, } = the square on half the line.

Prop. 9,

The squares on the unequal parts } = double the square on half the line, together with double the square on the line between points of section.

C.

Properties of a straight line bisected and produced.
Props. 6 and 10.



Let AB be a straight line bisected in C and produced to D.

Then, Prop. 6,

The rectangle contained by the whole produced line
and the part produced, together with the square on half
the line bisected = the square on the straight
line made up of the half
and the part produced.

Prop. 10,

The squares on the whole produced line and on the part produced = double the square on half the line bisected, together with double the square on the line made up of the half and the part produced.

SELECT GENERAL LISTS
OF
SCHOOL-BOOKS
PUBLISHED BY
MESSRS. LONGMANS AND CO.

The School-Books, Atlases, Maps, &c. comprised in this Catalogue may be inspected in the Educational Department of Messrs. LONGMANS and Co. 89 Paternoster Row, E.C. London, where also all other works published by them may be seen.

English Reading-Lesson Books.

| | |
|---|---------|
| Bilton's Infant Primer for School and Home use, 18mo. | 2d. |
| — Infant Reader, Narratives and Fables in Monosyllables, 18mo. ... | 4d. |
| — First Reading Book, for Standard I, 18mo. | 6d. |
| — Second Reading Book, for Standard II, 18mo. | 6d. |
| — Third Reading Book, <i>Boys' Edition</i> and <i>Girls' Edition</i> , fcp. 9d. each | 6d. |
| — Fourth Reading Book, <i>Boys' Edition</i> and <i>Girls' Edition</i> , fcp. 1s. each | 1s. |
| — Fifth Reading Book, or <i>Poetical Reader</i> , fcp. | 1s. 3d. |
| Isbister's First Steps in Reading and Learning, 18mo. | 1s. 6d. |
| — Word-Builder, First Standard, 6d. Second Standard, 8d. | |
| — Sixth Standard Reader, 18mo. | 1s. |
| Laurie & Morell's Graduated Series of Reading-Lesson Books — | |
| Morell's Elementary Reading Book or Primer, 18mo. | 2d. |
| Book I, pp. 144. | 8d. |
| Book II, pp. 254. | 1s. 3d. |
| Book III, pp. 312. | 1s. 6d. |
| Book IV, pp. 440. | 2s. |
| Book V, comprehending Readings in the best English Literature, pp. 496. | 2s. 6d. |
| M'Leod's Reading Lessons for Infant Schools, 30 Broadsides Sheets | 2s. |
| — First School-Book to teach Reading and Writing, 18mo. | 6d. |
| — Second School-Book to teach Spelling and Reading, 18mo. | 9d. |
| Stevens's Domestic Economy Series for Girls:— | |
| Book I, for Girls' Fourth Standard, crown 8vo. | 2s. |
| Book II, for Girls' Fifth Standard, crown 8vo. | 2s. |
| Book III, for Girls' Sixth Standard, crown 8vo. | 2s. |
| Stevens & Hole's Introductory Lesson-Book, 18mo. | 6d. |
| Stevens & Hole's Grade Lesson-Book Primer, crown 8vo. | 2d. |
| Stevens & Hole's Grade Lesson Books, in Six Standards, 12mo. :— | |
| The First Standard, pp. 128 ... 9d. The Fourth Standard, pp. 224 ... 1s. 8d. | |
| The Second Standard, pp. 160 ... 9d. The Fifth Standard, pp. 224 ... 1s. 8d. | |
| The Third Standard, pp. 160 ... 9d. The Sixth Standard, pp. 260 ... 1s. 6d. | |
| Answers to the Arithmetical Exercises in Standards I, II, and III, price 4d. in Standard IV, price 4d. in Standards V. and VI. 4d. or complete, price 1s. 2d. | |
| Stevens & Hole's Useful Knowledge Reading Books :— | |
| Boys' First Standard, 12mo. ... 6d. Girls' First Standard, 12mo. ... 6d. | |
| — Second Standard, 12mo. ... 8d. — Second Standard, 12mo. ... 8d. | |
| — Third Standard, 12mo. ... 9d. — Third Standard, 12mo. ... 9d. | |
| — Fourth Standard, 12mo. ... 1s. — Fourth Standard, 12mo. ... 1s. | |
| — Fifth Standard, 12mo. ... 1s. — Fifth Standard, 12mo. ... 1s. | |
| — Sixth Standard, 12mo. ... 1s. 2d. — Sixth Standard, 12mo. ... 1s. 2d. | |

London, LONGMANS & CO.

General Lists of School-Books

| | |
|--|----------|
| Jones's Secular Early Lesson-Book, 18mo. | 6d. |
| — Secular Early Lesson-Book, Part II. Proverbs | 4d. |
| — Advanced Reading-Book; Lessons in English History, 18mo. | 10d. |
| Marrot's Seasons, or Stories for Young Children, 4 vols. 18mo. | each 2s. |
| Sullivan's Literary Class-Book; Readings in English Literature, fop. | 2s. 6d. |

Writing Books.

| | |
|---|----------|
| Combe, Stevens, and Hole's Complete Writer: A Set of 16 Graduated Copy-Books, on Fine Paper, price 4s. 6d. per Dozen to Teachers. | |
| Johnson's Civil Service Specimens of Copying MSS. folio | 2s. 6d. |
| M'Leod's Graduated Series of Nine Copy-Books | each 3d. |
| Milhauser's Writing Books, 2s. 6d. per Dozen to Teachers. | |
| The Ready Writer, a Course of 18 Graduated Copy Books | each 2d. |
| Books I. to VIII. of the READY WRITER are printed in PENCIL-INK. | |

School Poetry Books.

| | | |
|--|--------------------------|--------------------------|
| Bilton's Poetical Reader for all Classes of Schools, fop. | 1s. 3d. | |
| Byron's Childe Harold, annotated by W. Hiley, M.A. fop. Svo. | 1s. 6d. | |
| Coleridge's Ancient Mariner, by Stevens & Morris, fop. 4d. sewed, 6d. cloth. | | |
| Cook's First Book of Poetry for Elementary Schools, 18mo. | 9d. | |
| Cowper's Task, <i>The Sofa</i> , by Stevens & Morris, fop. 9d. sewed, 1s. cloth. | | |
| Edwards's Poetry Book of Elder Poets, 18mo. | 2s. 6d. | |
| — Modern Poets, 18mo. | 2s. 6d. | |
| Goldsmith's Deserted Village, by Stevens & Morris, fop. 4d. sewed or 6d. cloth. | | |
| — Traveller, by Stevens & Morris, fop. Svo. 9d. sewed or 1s. cloth. | | |
| Gray's Elegy, edited by Stevens & Morris, fop. 4d. sewed, 6d. cloth. | | |
| Hughes' Select Specimens of English Poetry, 12mo. | 2s. 6d. | |
| Hunter's 33 Plays of Shakespeare, with Explanatory Notes, each Play 1s. | | |
| All's Well that ends Well. | Henry VI. Part III. | Much ado about Nothing. |
| Antony and Cleopatra. | Henry VIII. | Othello. |
| As You Like it. | Julius Caesar. | Richard II. |
| Comedy of Errors. | King John. | Richard III. |
| Coriolanus. | King Lear. | Romeo and Juliet. |
| Cymbeline. | Love's Labour's Lost. | Taming of the Shrew. |
| Hamlet. | Macbeth. | The Taming of the Shrew. |
| Henry IV. Part I. | Measure for Measure. | Timon of Athens. |
| Henry IV. Part II. | Merchant of Venice. | Troilus and Cressida. |
| Henry V. | Merry Wives of Windsor. | Twelfth-Night. |
| Henry VI. Part I. | Midsummer Night's Dream. | Two Gentlemen of Verona. |
| Henry VI. Part II. | | Winter's Tale. |

| | |
|--|---------|
| M'Leod's First Poetical Reading Book, fop. Svo. | 2d. |
| Second Poetical Reading Book, fop. Svo. | 1s. 6d. |
| M'Leod's Goldsmith's Deserted Village, and Traveller, each Poem, 18mo. | 1s. 6d. |
| Marlowe's Doctor Faustus, annotated by Wagner, fop. Svo. | 2s. |
| Milton's Lycidas, by Stevens & Morris, fop. 4d. sewed or 6d. cloth. | |
| — Samson Agonistes and Lycidas, by Hunter, 12mo. | 1s. 6d. |
| — L'Allegro, by Stevens & Morris, fop. 4d. sewed or 6d. cloth. | |
| — Il Penseroso, by Stevens & Morris, fop. 4d. sewed or 6d. cloth. | |
| — Comus, L'Allegro and Il Penseroso, by Hunter, 12mo. | |
| — Paradise Lost, by Hunter, I. & II. 1s. 6d. each; III. to V. 1s. each. | 1s. 6d. |
| — Paradise Regained, annotated by Herriman, fop. Svo. | 2s. 6d. |
| Pope's Essay on Man, annotated by Hunter, fop. Svo. | 1s. 6d. |
| — Select Poems, annotated by Arnold, fop. Svo. | 2s. 6d. |
| Scott's Lady of the Lake, Canto I. by Stevens & Morris, fop. 9d. sewed, 1s. cloth. | |
| Twells' Poetry for Repetition, comprising 200 short pieces, 18mo. | 2s. 6d. |

English Spelling-Books.

| | |
|---|---------|
| Johnson's Civil Service Spelling Book, fop. | 1s. 2d. |
| Bewell's Dictation Exercises, First Series, 18mo. 1s. Second Series | 2s. 6d. |
| Sullivan's Spelling-Book Superseded, 18mo. | 1s. 4d. |
| — Words Spelled in Two or More Ways, 18mo. | 1s. 6d. |

London, LONGMANS & CO.

General Lists of School-Books

3

Grammar and the English Language.

| | |
|---|----------------------|
| Arnold's English Authors, crown 8vo..... | <i>Nearly ready.</i> |
| — Manual of English Literature, crown 8vo. | 7s. 6d. |
| Bain's First or Introductory English Grammar, 18mo. | 1s. 6d. |
| — Higher English Grammar, fop. 8vo..... | 2s. 6d. |
| — Companion to English Grammar, crown 8vo..... | 2s. 6d. |
| Brewer's Guide to English Composition, fop. 8vo. | 5s. 6d. |
| Conway's Treatise on Verification, crown 8vo. | 4s. 6d. |
| Edwards's History of the English Language, with Specimens, 18mo. | 9d. |
| — (Miss) Prose-Book of Great English Writers, 16mo..... | 2s. 6d. |
| Farrar's Language and Languages, crown 8vo. | 6s. |
| Ferrar's Comparative Grammar, Sanskrit, Greek, Latin, VOL. I. 8vo. | 12s. |
| Fleming's Analysis of the English Language, crown 8vo. | 5s. |
| Goswick's English Grammar, Historical and Analytical, crown 8vo. | 10s. 6d. |
| Graham's English, or the Art of Composition Explained, fop. 8vo. | 5s. |
| Hiley's Child's First English Grammar, 18mo. | 1s. |
| Abridgment of Hiley's English Grammar, 18mo. | 1s. 9d. |
| Hiley's English Grammar and Style, 12mo. | 3s. 6d. |
| — Exercises adapted to his English Grammar, 18mo..... | 2s. 6d. Key 4s. 6d. |
| — Practical English Composition, Part I. 18mo. | 1s. 6d. Key 2s. 6d. |
| — — — — — Part II. 18mo. | 3s. Key 4s. |
| Hunter's Text-Book of English Grammar, 18mo. | 2s. 6d. |
| — Manual of School Letter-Writing, 18mo. | 1s. 6d. |
| Labister's English Grammar, 12mo. | 1s. 6d. |
| — First Book of Grammar, Geography, and History, 18mo. | 6d. |
| Johnston's English Composition and Essay-Writing, post 8vo. | 3s. 6d. |
| Letham's Handbook of the English Language, crown 8vo. | 6s. |
| — Elementary English Grammar, crown 8vo. | 2s. 6d. |
| — English Grammar for Classical Schools, fop. 8vo. | 2s. 6d. |
| — Outlines of Philology, crown 8vo. | 4s. 6d. |
| Lowres's Grammar of English Grammars, 12mo. | 3s. 6d. |
| — Companion to English Grammar, 18mo. | 2s. 6d. |
| M'Leod's Explanatory English Grammar for Beginners, 18mo. | 9d. |
| — English Grammatical Definitions, for Home Study, 18mo. | 1d. |
| Maracet's Willy's Grammar for the use of Boys, 18mo. | 2s. 6d. |
| — Mary's Grammar, intended for the use of Girls, 18mo. | 2s. |
| Morell's Essentials of English Grammar and Analysis, fop. 8vo. | 3d. |
| Morgan's Learner's Companion to the same, post 8vo. | 6d. |
| Morell's Grammar of the English Language, post 8vo. 2s. or with Exercises 2s. 6d. | |
| — Graduated English Exercises, post 8vo. 8d. sewed or 9d. cloth. | |
| Morgan's Key to Morell's Graduated Exercises, 18mo. | 4s. |
| Müller's (Max) Lectures on the Science of Language, 2 vols. crown 8vo. | 16s. |
| Murison's First Work in English, fop. 8vo. | 3s. 6d. |
| Roget's Thesaurus of English Words and Phrases, crown 8vo. | 10s. 6d. |
| The Stepping-Stone to English Grammar, 18mo. | 1s. |
| Sullivan's Manual of Etymology, or First Steps to English, 18mo. | 10d. |
| — Attempt to Simplify English Grammar, 18mo. | 1s. |
| Wadham's English Verification, crown 8vo. | 4s. 6d. |
| Weymouth's Answers to Questions on the English Language, fop. 8vo. | 2s. 6d. |
| Yonge's Short English Grammar, crown 8vo. | 2s. 6d. |

Paraphrasing, Parsing, and Analysis.

| | |
|--|---------------------|
| Hunter's Indexing & Précis of Correspondence, 12mo. | 3s. 6d. |
| — Introduction to Précis-Writing, 12mo. | 2s. |
| — Paraphrasing and Analysis of Sentences, 12mo. | 1s. 6d. Key 1s. 8d. |
| — Progressive Exercises in English Parsing, 12mo. | 6d. |
| — Questions on Paradise Lost, Books I. & II. 12mo. | 1s. |

London, LONGMANS & CO.

General Lists of School-Books

| | |
|--|---------|
| Hunter's Questions on the Merchant of Venice, 12mo. | 1s. |
| Johnston's Civil Service Précis, 12mo. | 2s. 6d. |
| Lowree's System of English Parsing and Derivation, 18mo. | 1s. |
| Morell's Analysis of Sentences Explained and Systematised, 12mo. | 2s. |
| Morgan's Training Examiner, First Course, 4d. Second Course, 1s. | |

Dictionaries; with Manuals of Etymology.

| | |
|---|---------|
| Black's Student's Manual of Words derived from the Greek, 18mo. | 1s. 6d. |
| — — — — Latin, 18mo. | 2s. 6d. |
| — Student's Manual, Greek and Latin, complete, 18mo. | 2s. 6d. |
| Graham's English Synonyms, Classified and Explained, fcp. 8vo. | 6s. |
| Latham's English Dictionary, founded on Dr. Johnson's, 4 vols. 4to. price | 5l. |
| — Abridged English Dictionary, 1 vol. medium 8vo. | 2s. 6d. |
| Maunder's Scientific and Literary Treasury, fcp. 8vo. | 6s. |
| — Treasury of Knowledge and Library of Reference, fcp. 8vo. | 6s. |
| Sullivan's Dictionary of the English Language, 12mo. | 2s. |
| — Dictionary of Derivations, or Introduction to Etymology, fcp.... | 2s. |
| Whately's English Synonyms, fcp. 8vo. | 2s. |

Elocution.

| | |
|--|-----------|
| Bilton's Repetition and Reading Book, crown 8vo. | 2s. 6d. — |
| Hughes's Select Specimens of English Poetry, 12mo. | 2s. 6d. — |
| Ibbister's Illustrated Public School Speaker and Reader, 12mo. | 2s. 6d. — |
| — Lessons in Elocution, for Girls, 12mo. | 1s. 6d. |
| — Outlines of Elocution, for Boys, 12mo. | 1s. 6d. — |
| Bowton's Debater, or Art of Public Speaking, fcp. 8vo. | 6s. |
| Smart's Practice of Elocution, 12mo. | 4s. |
| Twells's Poetry for Repetition, 200 short Pieces and Extracts, 18mo. | 2s. 6d. — |

The London Series of English Classics.

| | |
|---|-----------|
| Bacon's Essays, annotated by E. A. Abbott, D.D., 2 vols. fcp. | 6s. |
| — The same, Text and Index only, without Notes, 1 vol. | 2s. 6d. — |
| Ben Jonson's Every Man in his Humour, by H. B. Wheatley, F.S.A. | 2s. 6d. — |
| Macaulay's Essay on Lord Clive, annotated by H. C. Bowen, M.A. | 2s. 6d. — |
| Marlowe's Dr. Faustus, annotated by Wilhelm Wagner, Ph.D. | 2s. |
| Milton's Paradise Regained, annotated by C. S. Jerram, M.A. | 2s. 6d. — |
| Selections from Pope's Poems, annotated by T. Arnold, M.A. | 2s. 6d. — |

Arithmetic.

| | |
|--|---------|
| Anderson's Arithmetic for the Army, 18mo. | 1s. |
| Calder's Familiar Arithmetic, 12mo. 4s. 6d. or with Answers, 5s. 6d. the Answers separately, 1s. the Questions in Part II, separately. | 1s. |
| Calder's Smaller Arithmetic for Schools, 18mo. | 2s. 6d. |
| Colenso's Arithmetic designed for the use of Schools, 12mo. | 4s. 6d. |
| Key to Colenso's Arithmetic for Schools, by Rev. J. Hunter, M.A. 12mo. | 5s. |
| Colenso's Shilling Elementary Arithmetic, 18mo. 1s. with Answers | 1s. 6d. |
| — Arithmetic for National, Adult, and Commercial Schools:— | |
| 1. Text-Book, 18mo. 6d. 3. Examples, Part II. Compound Arithmetic 6d. | |
| 2. Examples, Part I. Simple 4. Examples, Part III. Fractions, Decimals Arithmetic 4d. Duodecimals 6d. | |
| 5. Answers to Examples, with Solutions of the difficult Questions ... 1s. | |
| Colenso's Arithmetical Tables, on a Card | 1d. |
| Combes and Hines' Standard Arithmetical Copy-Books, Nine Books, 4d. each. | |
| Combes and Hines' Complete Arithmetical Copy-Books; in Nine Books, on Fine Paper, 4d. to 6d. each. Price 4s. 6d. per dozen to Teachers. | |
| Harris's Graduated Exercises in Arithmetic and Mensuration, crown 8vo. 2s. 6d. or with Answers, 3s. the Answers separately, 9d. Full Key 6s. | |

London, LONGMANS & CO.

General Lists of School-Books

5

| | |
|---|-----------------|
| Hiley's Recapitulatory Examples in Arithmetic, 12mo. | 1s. 6d. |
| Hunter's Modern Arithmetic for School Work or Private Study, 12mo. 2s. 6d. Key, 5s. | |
| Hunter's New Shilling Arithmetic, 12mo. | 1s. Key 2s. |
| — Standard Arithmetic, Three Parts, 2d. each, and Key | 6d. |
| Ibbister's High School Arithmetic, 12mo. 1s. or with Answers | 1s. 6d. |
| Johnston's Civil Service Arithmetic, 12mo. | 2s. 6d. Key 4s. |
| — Civil Service Tots with Answers and Cross-Tots | 1s. |
| Addiss' Arithmetic, 12mo. 1s.—or Two Parts | each 6d. |
| Lupton's Arithmetic for Schools and Candidates for Examination, 12mo. | |
| — 2s. 6d. or with Answers, 3s. 6d. the Answers separately 1s. | Key 6s. |
| — Examination-Papers in Arithmetic, crown 8vo. | 1s. |
| M'Leod's Manual of Arithmetic, containing 1,750 Questions, 12mo. | 9d. |
| — Mental Arithmetic, I. Whole Numbers, II. Fractions | each 1s. |
| — Extended Multiplication and Pence Tables, 12mo. | 2d. |
| Merrifield's Technical Arithmetic and Mensuration, small 8vo. 2s. 6d. Key 3s. 6d. | |
| Moffatt's Mental Arithmetic, 12mo. 1s. or with Key, 1s. 6d. | |
| Pix's Miscellaneous Examples in Arithmetic, 12mo. | 2s. 6d. |
| Tate's First Principles of Arithmetic, 12mo. | 1s. 6d. |

Book-keeping and Banking.

| | |
|---|---------------------|
| Hunter's Exercises in Book-keeping by Double Entry, 12mo. | 1s. 6d. Key 2s. 6d. |
| — Examination-Questions in Book-keeping by Double Entry, 12mo. | 2s. 6d. |
| — Examination-Questions 2s. as above, separate from the Answers | 1s. |
| — Ruled Paper for Forms of Account Books, 5 sorts .. per quire, | 1s. 6d. |
| — Self-Instruction in Book-keeping, 12mo. | 2s. |
| — Studies in Double Entry, crown 8vo. | 2s. |
| Ibbister's Book-keeping by Single and Double Entry, 12mo. | 9d. |
| — Set of Eight Account Books to the above | each 6d. |
| Macleod's Economics for Beginners, small crown 8vo. | 2s. 6d. |
| — Elements of Banking, Fourth Edition, crown 8vo. | 5s. |

Mensuration.

| | |
|---|-------------|
| Boncher's Mensuration, Plane and Solid, 12mo. | 2s. |
| Hiley's Explanatory Mensuration, 12mo. | 2s. 6d. |
| Hunter's Elements of Mensuration, 12mo. | 1s. Key 9d. |
| Merrifield's Technical Arithmetic & Mensuration, small 8vo. | 2s. 6d. |
| Seablit's Treatise on Practical Mensuration, by Hunter, 12mo. 2s. 6d. Key 5s. | |

Algebra.

| | |
|---|---------------------|
| Coleman's Algebra, for National and Adult Schools, 12mo. | 1s. 6d. Key 2s. 6d. |
| — Algebra, for the use of Schools, Part I, 12mo | 4s. 6d. Key 5s. |
| — Elements of Algebra, for the use of Schools, Part II, 12mo. 6s. Key 5s. | |
| — Examples and Equation Papers, with the Answers, 12mo. | 2s. 6d. |
| — Student's Algebra, crown 8vo. | 6s. Key 6s. |
| Griffin's Algebra and Trigonometry, small 8vo. | 2s. 6d. |
| — Notes on Algebra and Trigonometry, small 8vo. | 2s. 6d. |
| Lund's Short and Easy Course of Algebra, crown 8vo. | 2s. 6d. Key 2s. 6d. |
| Lupton's Algebra for Army, &c. Examinations, 12mo. | 2s. 6d. |
| Reynolds's Elementary Algebra for Beginners, 12mo. 9d. Answers, 3d. Key 1s. | |
| Tate's Algebra made Easy, 12mo. | 2s. Key 2s. 6d. |
| Wood's Algebra, modernised by Lund, crown 8vo. | 7s. 6d. |
| — Companion to, by Lund, crown 8vo. | 7s. 6d. |

Geometry and Trigonometry.

| | |
|--|---------|
| Booth's New Geometrical Methods, 2 vols. 8vo. | 36s. |
| Coleman's Elements of Euclid, 12mo. 4s. 6d. or with Key to the Exercises ... | 6s. 6d. |
| — Geometrical Exercises and Key | 2s. 6d. |
| — Geometrical Exercises, separately, 12mo. | 1s. |
| — Trigonometry, 12mo. Part I. 3s. 6d. Key 3s. 6d. Part II. 2s. 6d. Key 5s. | |

London, LONGMANS & CO.

General Lists of School-Books

| | |
|--|----------------------|
| Griffin's Parabola, Ellipse, and Hyperbola, post 8vo. | 6s. |
| Harvey's Euclid for Beginners, Books I. & II. | <i>Nearly ready.</i> |
| Hawtrey's Introduction to Euclid | fcp. 8vo. 2s. 6d. |
| Hunter's Plane Trigonometry, for Beginners, 18mo. | 1s. Key 9d. |
| — Treatise on Logarithms, 18mo. | 1s. Key 9d. |
| Ibsister's School Euclid, 12mo. Book I. price 1s. Books I. & II. price 1s. 6d. | |
| Books I. to IV. price 2s. 6d. | |
| Jeans's Plane and Spherical Trigonometry, 18mo. Part I. 5s., Part II. 4s. | |
| or the 2 Parts in 1 vol. price 8s. 6d. | |
| Potts's Euclid. University Edition, 8vo. | 10s. |
| — Intermediate Edition, Books I. to IV. 3s. Books I. to III. 2s. 6d. | |
| Books I. II. 1s. 6d. Book I. 1s. | |
| — Enunciations of Euclid, 12mo. | 6d. |
| Salmon on Conic Sections, 6th Edition, 8vo. | 12s. |
| Tate's Differential and Integral Calculus, 18mo. | 4s. 6d. |
| — First Three Books of Euclid, 18mo. | 9d. |
| — Practical Geometry, with 261 Woodcuts, 18mo. | 1s. |
| — Geometry, Mensuration, Trigonometry, &c. 18mo. | 2s. 6d. |
| Thomson's Euclid, Books I. to VI. and XI. & XII. 12mo. | 5s. |
| — Plane and Spherical Trigonometry, 8vo. | 4s. 6d. |
| — Differential and Integral Calculus, 12mo. | 5s. 6d. |
| Watson's Plane and Solid Geometry, small 8vo. | 2s. 6d. |
| Williamson on Differential Calculus, crown 8vo. | 10s. 6d. |
| — on Integral Calculus, crown 8vo. | 10s. 6d. |
| Wilcock's Elementary Geometry of the Right Line, crown 8vo. | 5s. |
| Wright's Elements of Plane Geometry, crown 8vo. | 5s. |

Land Surveying, Drawing, and Practical Mathematics.

| | |
|---|----------|
| Binn's Orthographic Projection and Isometrical Drawing, 18mo. | 1s. |
| Kimber's Mathematical Course for the University of London, 8vo. | 12s. |
| Part I. for Matriculation, separately, 1s. 6d. Key, in 2 Parts, 5s. each. | |
| Nesbit's Practical Land Surveying, 8vo. | 12s. |
| Pierse's Solid or Descriptive Geometry, post 8vo. | 12s. 6d. |
| Salmon's Treatise on Conic Sections, 8vo. | 12s. |
| Winter's Mathematical Exercises, post 8vo. | 6s. 6d. |
| Winter's Elementary Geometrical Drawing, Part I. post 8vo. 2s. 6d. Part II. 1s. 6d. | |
| Wrigley's Examples in Pure and Mixed Mathematics, 8vo. | 6s. 6d. |

Musical Works by John Hullah, LL.D.

| | |
|--|----------------|
| Chromatic Scale, with the Inflected Syllables, on Large Sheet | 1s. 6d. |
| Card of Chromatic Scale, price 1d. | |
| Exercises for the Cultivation of the Voice. For Soprano or Tenor | 2s. 6d. |
| Grammar of Musical Harmony, royal 8vo. Two Parts | each 1s. 6d. |
| Exercises to Grammars of Musical Harmony | 1s. |
| Grammar of Counterpoint. Part I. super-royal 8vo. | 2s. 6d. |
| Hullah's Manual of Singing. Parts I. & II. 2s. 6d. or together | 5s. |
| Exercises and Figures contained in Parts I. & II. Books I. & II. | each 2d. |
| Large Sheets, containing the Figures in Part I. Nos. 1 to 8 in a Parcel | 6s. |
| Large Sheets, containing the Exercises in Part I. Nos. 9 to 40, in Four Parcels of Eight Nos. each | per Parcel 6s. |
| Large Sheets, the Figures in Part II. Nos. 41 to 52 in a Parcel | 9s. |
| Hymns for the Young, set to Music, royal 8vo. | 2d. |
| Infant School Songs | 6d. |
| Notation, the Musical Alphabet, crown 8vo. | 6d. |
| Old English Songs for Schools, Harmonised | 6d. |
| Rudiments of Musical Grammar, royal 8vo. | 2s. |
| School Songs for 2 and 3 Voices. 2 Books, 8vo. | each 6d. |
| Time and Tune in the Elementary School, crown 8vo. | 2s. 6d. |
| Exercises and Figures in the same, crown 8vo. 1s. or 2 Parts, 6d. each. | |

London, LONGMANS & CO.

Political and Historical Geography.

| | |
|--|----------------------|
| ury's Mary's Geography, 18mo. 2s. 6d. | Questions 1s. |
| ur's Public Schools Manual of Modern Geography | <i>In the press.</i> |
| - Ancient and Modern Geography, post 8vo. | 7s. 6d. |
| - Sketch of Modern Geography, post 8vo. | 4s. |
| - Sketch of Ancient Geography, post 8vo. | 4s. |
| r's Child's First Geography, 18mo. | 9d. |
| Elementary Geography for Beginners, 18mo. | 1s. 6d. |
| Compendium of European Geography and History, 18mo | 3s. 6d. |
| Asiatic, African, American and Australian Geography, 18mo. | 3s. |
| hes's Child's First Book of Geography, 18mo. | 9d. |
| - Geography of the British Empire, for Beginners, 18mo. | 9d. |
| - General Geography, for Beginners, 18mo. | 9d. |
| tions on Hughes's General Geography, for Beginners, 18mo. | 9d. |
| hes's Geography of British History, fcp. 8vo. | 5s. |
| - Manual of Geography, with Six Coloured Maps, fcp. 8vo. | 7s. 6d. |
| In Two Parts—I. Europe, 3s. 6d. II. Asia, Africa, America, &c. | 4s. |
| he's Manual of British Geography, fcp. 8vo. | 2s. |
| h Johnston's Gazetteer, or Geographical Dictionary, 8vo | 42s. |
| on's Examination-Papers in Geography, crown 8vo. | 1s. |
| sod's Geography of Palestine or the Holy Land, 18mo. | 1s. 6d. |
| nder's Treasury of Geography, fcp. 8vo. | 6s. |
| Stepping-Stone to Geography, 18mo. | 1s. |
| van's Geography Generalised, fcp. 2s. or with Maps, 2s. ed. | 1s. |
| - Introduction to Ancient and Modern Geography, 18mo. | 1s. |

Physical Geography and Geology.

| | |
|--|---------------------|
| o's Rocks Classified and Described, by Lawrence, post 8vo. | 14s. |
| he's (E.) Outlines of Physical Geography, 12mo. 3s. 6d. | Questions, 6d. |
| - (W.) Physical Geography for Beginners, 18mo. | 1s. |
| h's Treatise on the Use of the Globes, 12mo. | 6s. 6d. Key 2s. 6d. |
| ry's Physical Geography for Schools and General Readers, fcp. 8vo. | 2s. 6d. |
| le's Puzzle of Life (Elementary Geology), crown 8vo. | 3s. 6d. |
| tor's Elementary Physical Geography, fcp. 8vo. | 1s. 6d. |
| dward's Geology of England and Wales, crown 8vo. | 14s. |

School Atlases and Maps.

| | |
|---|----------|
| er's Atlas of Modern Geography, royal 8vo. | 10s. 6d. |
| - Junior Modern Atlas, comprising 12 Maps, royal 8vo. | 4s. 6d. |
| - Atlas of Ancient Geography, royal 8vo. | 12s. |
| - Junior Ancient Atlas, comprising 12 Maps, royal 8vo. | 4s. 6d. |
| - General Atlas, Modern & Ancient, royal 4to. | 22s. |
| le Schools Atlas of Ancient Geography, 2s entirely New Coloured Maps, imperial 8vo. or imperial 4to. 7s. 6d. cloth. | |
| le Schools Atlas of Modern Geography, 3s entirely New Coloured Maps, imperial 8vo. or imperial 4to. 5s. cloth. | |

Natural History and Botany.

| | |
|--|----------|
| iley and Moore's Treasury of Botany, Two Parts, fcp. 8vo. | 12s. |
| alister's Systematic Zoology of Vertebrates, 8vo. | 10s. 6d. |
| nder's Treasury of Natural History, revised by Holdsworth, fcp. 8vo. 6s. | |
| m's Natural History for Beginners, 18mo. Two Parts 9d. each, or 1 vol. 2s. | |
| - Stepping-Stone to Natural History, 18mo. | 3s. 6d. |
| Or in Two Parts.—I. <i>Mammalia</i> , 1s. II. <i>Birds, Reptiles, and Fishes</i> 1s. | |

General Lists of School-Books

| | |
|---------------------------------|---------|
| Wood's Bible Animals, 8vo. | 14s. |
| — Homes without Hands, 8vo. | 14s. |
| — Insects at Home, 8vo. | 14s. |
| — Insects Abroad, 8vo. | 14s. |
| — Out of Doors, crown 8vo. | 7s. 6d. |
| — Strange Dwellings, crown 8vo. | 7s. 6d. |

Chemistry and Telegraphy.

| | |
|--|----------|
| Armstrong's Organic Chemistry, small 8vo. | 2s. 6d. |
| Crookes's Select Methods in Chemical Analysis, crown 8vo. | 12s. 6d. |
| Challay's Practical Telegraphy, 8vo. | 14s. |
| Miller's Elements of Chemistry, 3 vols. 8vo. | |
| Part I.—Chemical Physics, Sixth Edition, 16s. | |
| Part II.—Inorganic Chemistry, Sixth Edition, 24s. | |
| Part III.—Organic Chemistry, Sixth Edition in the press. | |
| — Introduction to Inorganic Chemistry, small 8vo. | 2s. 6d. |
| Odling's Course of Practical Chemistry, for Medical Students, crown 8vo. | 6s. |
| Preece and Sivewright's Telegraphy, crown 8vo. | 2s. 6d. |
| Tate's Outline of Experimental Chemistry, 18mo. | 2d. |
| Thorpe's Quantitative Chemical Analysis, small 8vo. | 4s. 6d. |
| Thorpe and Muir's Qualitative Chemical Analysis, small 8vo. | 2s. 6d. |
| Tilden's Theoretical and Systematic Chemistry, small 8vo. | 2s. 6d. |

Natural Philosophy and Natural Science.

| | |
|--|----------|
| Bloxam's Metals, their Properties and Treatment, small 8vo. | 2s. 6d. |
| Day's Numerical Examples in Heat, crown 8vo. | 1s. 6d. |
| — Electrical & Magnetic Measurement, 16mo. | 2s. 6d. |
| Downing's Practical Hydraulics, Part I. 8vo. | 5s. 6d. |
| Ganot's Physics, translated by Prof. E. Atkinson, large crown 8vo. | 15s. |
| — Natural Philosophy, translated by the same, crown 8vo. | 7s. 6d.— |
| Gore's Art of Scientific Discovery, crown 8vo. | 15s. |
| Heimholz's Popular Lectures on Scientific Subjects, 8vo. | 12s. 6d. |
| Irving's Short Manual of Heat, small 8vo. | 2s. 6d. |
| Jenkin's Electricity & Magnetism, small 8vo. | 2s. 6d. |
| Marcet's Conversations on Natural Philosophy, 12mo. 8vo. | 7s. 6d. |
| Maxwell's Theory of Heat, small 8vo. | 2s. 6d. |
| Merrifield's Natural Science Reading Books. | |
| Minchin's Treatise on Statics, crown 8vo. | 10s. 6d. |
| Tate's Light & Heat, for the use of Beginners, 18mo. | 9d. |
| — Hydrostatics, Hydraulics & Pneumatics, 18mo. | 9d. |
| — Electricity, explained for the use of Beginners, 18mo. | 9d. |
| — Magnetism, Voltaic Electricity & Electro-Dynamics, 18mo. | 9d. |
| Tyndall's Lesson in Electricity, with 55 Woodcuts, crown 8vo. | 2s. 6d. |
| — Notes of Lectures on Electricity, 1s. sewed, 1s. 6d. cloth. | |
| — Notes of Lectures on Light, 1s. sewed, 1s. 6d. cloth. | |
| Weinhold's Introduction to Experimental Physics, 8vo. | 11s. 6d. |

Text-Books of Science, Mechanical and Physical.

| | |
|--|---------|
| Abney's Treatise on Photography, small 8vo. | 2s. 6d. |
| Anderson's (Sir John) Strength of Materials. | 3s. 6d. |
| Armstrong's Organic Chemistry. | 2s. 6d. |
| Barry's Railway Appliances. | 2s. 6d. |
| Bloxam's Metals. | 2s. 6d. |
| Goodeve's Elements of Mechanism. | 2s. 6d. |
| — Principles of Mechanics. | 2s. 6d. |
| Gore's Art of Electro-Metallurgy. | 6s. |
| Griffin's Algebra and Trigonometry. | 2s. 6d. |

General Lists of School-Books

9

| | |
|---|---------|
| Jenkin's Electricity and Magnetism | 2s. 6d. |
| Maxwell's Theory of Heat | 2s. 6d. |
| Merrifield's Technical Arithmetic and Mensuration | 2s. 6d. |
| Miller's Inorganic Chemistry | 2s. 6d. |
| Preece & Sivewright's Telegraphy | 2s. 6d. |
| Rutley's Study of Rocks, a Text-Book of Petrology | 4s. 6d. |
| Shelley's Workshop Appliances | 2s. 6d. |
| Thomé's Structural and Physiological Botany | 6s. |
| Thorpe's Quantitative Chemical Analysis | 4s. 6d. |
| Thorpe & Muir's Qualitative Analysis | 2s. 6d. |
| Tilden's Chemical Philosophy | 2s. 6d. |
| Unwin's Elements of Machine Design | 2s. 6d. |
| Watson's Plane and Solid Geometry | 2s. 6d. |

The London Science Class-Books, Elementary Series.

| | |
|--|---------|
| Astronomy, by R. S. Ball, LL.D. F.R.S. | 1s. 6d. |
| Botany, Morphology and Physiology, by W. R. McNab, M.D. | 1s. 6d. |
| — the Classification of Plants, by W. R. McNab, M.D. | 1s. 6d. |
| Geometry, Congruent Figures, by O. Henrici, F.R.S. | 1s. 6d. |
| Hydrostatics and Pneumatics, by P. Magnus, B.Sc. 1s. 6d. or with Answers 2s. | |
| Mechanics, by R. S. Ball, LL.D. F.R.S. | 1s. 6d. |
| Practical Physics : Molecular Physics & Sound, by F. Guthrie, F.R.S. | 1s. 6d. |
| Thermodynamics, by R. Wormell, M.A. D.Sc. | 1s. 6d. |
| Zoology of Vertebrate Animals, by A. McAlister, M.D. | 1s. 6d. |
| Zoology of Invertebrate Animals, by A. McAlister, M.D. | 1s. 6d. |

Mechanics and Mechanism.

| | |
|--|-----------------|
| Barry's Railway Appliances, small 8vo. Woodcuts..... | 2s. 6d. |
| Goodewe's Elements of Mechanism, small 8vo. | 2s. 6d. |
| — Principles of Mechanics, small 8vo. | 2s. 6d. |
| Haughton's Animal Mechanics, 8vo. | 2s. 6d. |
| Magnus's Lessons in Elementary Mechanics, small 8vo. | 2s. 6d. |
| Shelley's Workshop Appliances, small 8vo. Woodcuts | 2s. 6d. |
| Tate's Exercises on Mechanics and Natural Philosophy, 12mo. | 2s. Key 2s. 6d. |
| — Mechanics and the Steam-Engine, for Beginners, 18mo. | 9d. |
| Twissden's Introduction to Practical Mechanics, crown 8vo. | 10s. 6d. |
| — First Lessons in Theoretical Mechanics, crown 8vo. | 8s. 6d. |
| Willis's Principles of Mechanism, 8vo. | 12s. |

Engineering, Architecture, &c.

| | |
|---|----------|
| Anderson on the Strength of Materials and Structures, small 8vo. | 2s. 6d. |
| Bourne's Treatise on the Steam-Engine, 4to. | 42s. |
| — Catechism of the Steam-Engine, 12mo. | 6s. |
| — Recent Improvements in the Steam-Engine, 12mo. | 6s. |
| — Handbook of the Steam-Engine, 12mo. | 6s. |
| Downing's Elements of Practical Construction, PART I, 8vo. Plates | 12s. |
| Fairbairn's Mills and Millwork, 1 vol. 8vo. | 22s. |
| — Useful Information for Engineers. 3 vols., crown 8vo. | 31s. 6d. |
| Gwilt's Encyclopaedia of Architecture, 8vo. | 52s. 6d. |
| Main and Brown's Marine Steam-Engine, 8vo. | 12s. 6d. |
| — Indicator & Dynamometer, 8vo. | 4s. 6d. |
| — Questions on the Steam-Engine, 8vo. | 6s. 6d. |
| Mitchell's Stepping-Stone to Architecture, 18mo. Woodcuts | 1s. |

London, LONGMANS & CO.

Popular Astronomy and Navigation.

| | |
|--|---------|
| Brinkley's Astronomy, by Stables & Brinklow, crown 8vo. | 6s. |
| Evans's Navigation & Great Circle Sailing, 18mo. | 1s. |
| Herschel's Outlines of Astronomy, Twelfth Edition, square crown 8vo. | 1s. |
| Jones's Handbook for the Stars, royal 8vo. | 4s. 6d. |
| — Navigation and Nautical Astronomy, royal 8vo. Practical, 7s. 6d. Part II. Theoretical, 7s. 6d. or the 3 Parts in 1 vol. price 1s. | |
| Leighton's Naval Surveying, small 8vo. | 6s. |
| Merrifield's Magnetism & Deviations of the Compass, 18mo. | 1s. 6d. |
| Proctor's Lessons in Elementary Astronomy, fcp. 8vo. | 1s. 6d. |
| — Library Star Atlas, folio. | No. |
| — New Star Atlas for Schools, crown 8vo. | 6s. |
| — Handbook for the Stars, square fcp. 8vo. | 6s. |
| The Stepping-Stone to Astronomy, 18mo. | 1s. |
| Tate's Astronomy and the use of the Globes, for Beginners, 18mo. | 9d. |
| Webb's Celestial Objects for Common Telescopes, New Edition in preparation. | |

Animal Physiology and Preservation of Health.

| | |
|---|------------------|
| Bray's Education of the Feelings, crown 8vo. | 2s. 6d. |
| — Physiology and the Laws of Health, 11th Thousand, fcp. 8vo. | 1s. 6d. |
| — Diagrams for Class Teaching. | per pair 6s. 6d. |
| Buckton's Food and Home Cookery, crown 8vo. | 2s. |
| — Health in the House, crown 8vo. | 2s. |
| — Town & Window Gardening, crown 8vo. | 2s. |
| Hartley's Air and its Relations to Life, small 8vo. | 6s. |
| Houses I Live In ; Structure and Functions of the Human Body, 18mo. | 2s. 6d. |
| Magothorpe's Animal Physiology, 18mo. | 1s. |

General Knowledge and Chronology.

| | |
|---|----------|
| Crook's Events of England in Rhyme, square 16mo. | 1s. |
| Sister's <i>Sententiae Chronologicae</i> , the Original Work, 18mo. | 1s. 6d. |
| — — — improved by Miss Sewell, 18mo. | 1s. 6d. |
| Stepping-Stone (The) to Knowledge, 18mo. | 1s. |
| Second Series of the Stepping-Stone to General Knowledge, 18mo. | 1s. |
| Sterne's Questions on Generalities, Two Series, each 2s. Keys | each 4s. |

Mythology and Antiquities.

| | |
|---|---------|
| Becker's <i>Gallus</i> , Roman Scenes of the Time of Augustus, post 8vo. | 7s. 6d. |
| — <i>Chariotole</i> , Illustrating the Private Life of the Ancient Greeks ... | 7s. 6d. |
| Ewald's Antiquities of Israel, translated by Solly, 8vo. | 1s. 6d. |
| Hort's New Pantheon, 18mo. with 17 Plates. | 2s. 6d. |
| Rich's Illustrated Dictionary of Roman and Greek Antiquities, post 8vo. | 7s. 6d. |

Biography.

| | |
|--|-----|
| Gleig's Life of the Duke of Wellington, crown 8vo. | 6s. |
| Jones's Life of Sir Martin Frobisher, crown 8vo. | 6s. |
| Macaulay's Oliver, annotated by H. C. Bowen, M.A. fcp. 8vo. | 6s. |
| Maurand's Biographical Treasury, re-written by W. L. B. Oates, fcp. 8vo. | 6s. |
| Stepping-Stone (The) to Biography, 18mo. | 1s. |

Epochs of Modern History.

| | |
|---|-----------------|
| Church's Beginning of the Middle Ages, fcp. 8vo. Maps | 2s. 6d. |
| Cordery's French Revolution to the Battle of Waterloo | In preparation. |
| Cox's Crusades, fcp. 8vo. Maps | 2s. 6d. |
| Creighton's Age of Elizabeth, fcp. 8vo. Maps | 2s. 6d. |
| Gairdner's Houses of Lancaster & York, fcp. 8vo. Maps | 2s. 6d. |

| | |
|--|------------------------|
| Gardiner's Thirty Years' War, 1618-1648, fcp. 8vo. Maps | 2s. 6d. |
| — First Two Stuarts and the Puritan Revolution, fcp. 8vo. Maps | 2s. 6d. |
| Hale's Fall of the Stuarts, fcp. 8vo. Maps | 2s. 6d. |
| Johnson's Normans in Europe, fcp. 8vo. Maps | 2s. 6d. |
| Longman's Frederick the Great and the 7 Years' War..... | <i>In preparation.</i> |
| Ladlow's War of American Independence, fcp. 8vo. Maps | 2s. 6d. |
| McCarthy's Epoch of Parliamentary Reform | <i>In preparation.</i> |
| Moberly's Early Tudors | <i>In preparation.</i> |
| Morris's Age of Anne, cp. 8vo. Maps | 2s. 6d. |
| Seeboldin's Protestant Revolution, fcp. 8vo. Maps | 2s. 6d. |
| Stubbs's Early Plantagenets, fcp. 8vo. Maps | 2s. 6d. |
| — Empire under the House of Hohenstaufen | <i>In preparation.</i> |
| Warburton's Edward the Third, fcp. 8vo. Maps | 2s. 6d. |

Epochs of English History.

| | |
|--|-----|
| Creighton's Shilling History of England, Introductory, . 8vo..... | 8s. |
| Browning's Modern England, from 1820 to 1876, . 8vo..... | 9d. |
| Cordery's Struggle against Absolute Monarchy, 1603-1688, fcp. Maps | 9d. |
| Creighton's England a Continental Power, 1066-1216, fcp. Maps | 9d. |
| — Tudors and the Reformation, 1485-1603, fcp. 8vo. Maps | 9d. |
| Powell's Early England up to the Norman Conquest, fcp. 8vo. Maps | 1s. |
| Rowley's Rise of the People and Growth of Parliament, 1215-1485, fcp. Maps | 9d. |
| — Settlement of the Constitution, 1688-1776, fcp. Maps | 9d. |
| Tancock's England during the Revolutionary Wars, 1778-1830, | 9d. |
| Epochs of English History, complete in 1 vol. fcp. 8vo. | 5s. |

British History.

| | |
|---|----------|
| Armitage's Childhood of the English Nation, fcp. 8vo. | 2s. 6d. |
| Bartle's Synopsis of English History, fcp. 8vo | 2s. 6d. |
| Cantlay's English History Analysed, fcp. 8vo | 2s. |
| Catechism of English History, edited by Miss Sewell, 18mo. | 1s. 6d. |
| Epochs of English History, edited by Creighton, fcp. 8vo. | 5s. |
| Geirdner's Richard III, and Perkin Warbeck, crown 8vo. | 10s. 6d. |
| Gleig's School History of England, abridged, 18mo. | 6s. |
| — First Book of History—England, 18mo. 2s. or 3 Parts, 9d. each. | |
| — British Colonies, or Second Book of History, 18mo. | 1s. |
| — British India, or Third Book of History, 18mo. | 9d. |
| — Historical Questions on the above Three Histories, 18mo. | 9d. |
| Littlewood's Essentials of English History, fcp. 8vo. | 2s. |
| Lupton's Examination-Papers in History, crown 8vo | 1s. |
| — English History, revised, crown 8vo. | 7s. 6d. |
| Macaulay's History of England, Student's Edition, 2 vols. crown 8vo. | 12s. |
| Morris's Class-Book History of England, fcp. 8vo. | 2s. 6d. |
| The Stepping-Stone to English History, 18mo. | 1s. |
| The Stepping-Stone to Irish History, 18mo. | 1s. |
| Turner's Analysis of English and French History, fcp. 8vo. | 2s. 6d. |

Epochs of Ancient History.

| | |
|---|---------|
| Beeby's Gracchi, Marius and Sulla, fcp. 8vo. Maps | 2s. 6d. |
| Casper's Age of the Antonines, fcp. 8vo. Maps | 2s. 6d. |
| — Early Roman Empire, fcp. 8vo. Maps | 2s. 6d. |
| Cox's Athenian Empire, fcp. 8vo. Maps | 2s. 6d. |
| — Greeks & Persians, fcp. 8vo. Maps | 2s. 6d. |
| Curteis's Rise of the Macedonian Empire, fcp. 8vo. Maps | 2s. 6d. |
| Ihne's Rome to its Capture by the Gauls, fcp. 8vo. Maps | 2s. 6d. |
| Merivale's Roman Triumvirates, fcp. 8vo. Maps | 2s. 6d. |
| Sankey's Spartan and Theban Supremacies, fcp. 8vo. Maps | 2s. 6d. |

History, Ancient and Modern.

| | |
|--|---------|
| Browne's History of Greece, for Beginners, 18mo. | 2s. |
| — History of Rome, for Beginners, 18mo. | 2s. |
| Grieg's History of France, 18mo. | 1s. |
| Irons' Roman History, Vols. I. to III. 8vo. | 4s. |
| Mackie's English Battles of the Peninsula, fcp. 8vo. | 1s. |
| Mangnall's Historical and Miscellaneous Questions, 18mo. | 4s. 6d. |
| Menzel's Historical Treasury, with Index, fcp. 8vo. | 6s. |
| Mervile's History of the Romans under the Empire, 8 vols. post 8vo. | 6s. |
| — Fall of the Roman Republic, 18mo. | 7s. 6d. |
| — General History of Rome, crown 8vo. Maps. | 7s. 6d. |
| Puller's School History of Rome, abridged from Mervile, fcp. Maps | 2s. 6d. |
| Rawlinson's Sixth Oriental Monarchy (the Parthians), 8vo. Maps &c. | 1s. |
| — Seventh Oriental Monarchy (the Sassanians) 8vo. Maps &c. 8vo. | 3s. |
| Sewell's Ancient History of Egypt, Assyria, and Babylonia, fcp. 8vo. | 6s. |
| — Catechism of Grecian History, 18mo. | 1s. 6d. |
| — Child's First History of Rome, fcp. 8vo. | 1s. 6d. |
| — First History of Greece, fcp. 8vo. | 2s. 6d. |
| — Popular History of France, crown 8vo. Maps | 7s. 6d. |
| Smith's Cathage and the Carthaginians, crown 8vo. | 1s. 6d. |
| The Stepping-Stone to Grecian History, 18mo. | 1s. |
| The Stepping-Stone to Roman History, 18mo. | 1s. |
| Taylor's Student's Manual of Ancient History, crown 8vo. | 7s. 6d. |
| — Student's Manual of Modern History, crown 8vo. | 7s. 6d. |
| Turner's Analysis of the History of Greece, fcp. 8vo. | 2s. 6d. |
| — Analysis of Roman History, fcp. 8vo. | 2s. 6d. |

Scripture History, Moral and Religious Works.

| | |
|---|----------|
| Ayre's Treasury of Bible Knowledge, fcp. 8vo. | 6s. |
| Boultbee's Commentary on the Thirty-Nine Articles, crown 8vo. | 6s. |
| Browne's Exposition of the Thirty-Nine Articles, 8vo. | 6s. |
| Examination Questions on the above, fcp. 8vo. | 2s. 6d. |
| Conder's Handbook to the Bible, post 8vo. Maps, &c. | 7s. 6d. |
| Conybeare and Howson's Life and Epistles of St. Paul, 1 vol. crown 8vo. | 9s. |
| Drummond's Jewish Messiah, 8vo. | 1s. |
| Grieg's Sacred History, or Fourth Book of History, 18mo. 2s. or 2 Parts, each | 9d. |
| Kalisch's Bible Studies, Part I. the Prophecies of Balaam, 8vo. | 10s. 6d. |
| Part II. the Book of Jonah | 10s. 6d. |
| Kalisch's Commentary on the Old Testament, with a New Translation. | |
| Vol. I. Genesis, 8vo. 1s. or adapted for the General Reader, 1s. Vol. II. | |
| Exodus, 1s. or adapted for the General Reader, 1s. Vol. III. | |
| Leviticus, Part I. 1s. or adapted for the General Reader, 1s. Vol. IV. | |
| Leviticus, Part II. 1s. or adapted for the General Reader, 1s. | |
| Morris's Catechist's Manual, 18mo. | 1s. 6d. |
| Potts' Paley's Evidences and Horae Paulinae, 8vo. | 10s. 6d. |
| Pullifrank's Teacher's Handbook of the Bible, crown 8vo. | 2s. 6d. |
| Riddle's Manual of Scripture History, fcp. 8vo. | 4s. |
| — Outline of Scripture History, fcp. 8vo. | 2s. 6d. |
| Roger's School and Children's Bible, crown 8vo. | 2s. |
| Rotch's History and Literature of the Israelites, 2 vols. crown 8vo.... | 12s. 6d. |
| — — — — — Abridged, fcp. 8vo... 2s. 6d. | |
| Sewell's Preparation for the Holy Communion, 32mo. | 2s. |
| The Stepping-Stone to Bible Knowledge, 18mo. | 1s. |
| Whately's Introductory Lessons on Christian Evidences, 18mo. | 6d. |

Mental and Moral Philosophy, and Civil Law.

| | |
|---|----------|
| Ames's Fifty Years of the British Constitution | 10s. 6d. |
| — Science of Jurisprudence, 8vo. | 10s. |
| — Primer of English Constitution and Government, crown 8vo. | 6s. |

London, LONGMANS & CO.

| | |
|--|----------|
| Bacon's Essays, with Annotations by Archbiishop Whately, 8vo. | 10s. 6d. |
| — — annotated by Hunter, crown 8vo. | 2s. 6d. |
| — — annotated by Abbott, 2 vols. fcp. 8vo. | 6s. |
| — — with References and Notes by Markby, fcp. 8vo. | 1s. 6d. |
| Bain's Rhetoric and English Composition, crown 8vo. | 4s. |
| — Mental and Moral Science, crown 8vo. | 10s. 6d. |
| Hume's Treatise on Human Nature, by Green and Grose, 2 vols. 8vo. | 22s. |
| — Essays, by the same Editors, 2 vols. 8vo. | 22s. |
| Lealie's Political and Moral Philosophy, 8vo. | 10s. 6d. |
| Lewis's History of Philosophy from Thales to Comte, 2 vols. 8vo. | 32s. |
| Lewis's Influence of Authority in Matters of Opinion, 8vo. | 1s. |
| Mill's System of Logic, Ratiocinative and Inductive, 2 vols. 8vo. | 22s. |
| Killick's Student's Handbook of Mill's System of Logic, crown 8vo. | 2s. 6d. |
| Morrell's Handbook of Logic, for Schools and Teachers, fcp. 8vo. | 2s. |
| Sandars's Institutes of Justinian, 8vo. | 1s. |
| Stobbing's Analysis of Mill's Logic, 18mo. | 2s. 6d. |
| Swinburne's Picture Logic, crown 8vo. | 5s. |
| Thomson's Outline of the Necessary Laws of Thought, post 8vo. | 6s. |
| Ueberweg's Logic, translated by Lindsay, 8vo. | 1s. |
| Whately's Elements of Logic, 8vo. 10s. 6d. crown 8vo. | 4s. 6d. |
| — Elements of Rhetoric, 8vo. 10s. 6d. crown 8vo. | 4s. 6d. |
| — Lessons on Reasoning, fcp. 8vo. | 1s. 6d. |

Principles of Teaching, &c.

| | |
|---|----------|
| Gill's Text-Book of Education, Method and School Management, fcp. 8vo. | 2s. |
| — Systems of Education, fcp. 8vo. | 2s. 6d. |
| — Art of Religious Instruction, fcp. 8vo. | 2s. |
| — Art of Teaching to Observe and Think, fcp. 8vo. | 2s. |
| Johnston's (Miss) Ladies' College and School Examiner, fcp. 1s. 6d. Key | 2s. 6d. |
| Johnston's (R.) Army and Civil Service Guide, crown 8vo. | 2s. |
| — Civil Service Guide, crown 8vo. | 2s. 6d. |
| — Guide to Candidates for the Excise, 18mo. | 1s. |
| — Guide to Candidates for the Customs, 18mo. | 1s. |
| Lake's Book of Oral Object Lessons on Common Things, 18mo. | 1s. 6d. |
| Potts's Liber Cantabrigiensis, fcp. 8vo. | 2s. 6d. |
| — Account of Cambridge Scholarships and Exhibitions, fcp. 8vo. | 1s. 6d. |
| — Maxims, Aphorisms, &c. for Learners, crown 8vo. | 1s. 6d. |
| Robinson's Manual of Method and Organisation, fcp. 8vo. | 2s. 6d. |
| Sewell's Principles of Education, 2 vols. fcp. 8vo. | 12s. 6d. |
| Sullivan's Papers on Education and School-Keeping, 18mo. | 2s. |

The Greek Language.

| | |
|---|---------|
| Bloomfield's College and School Greek Testament, fcp. 8vo. | 6s. |
| Bolland & Lang's Politics of Aristotle, post 8vo. | 7s. 6d. |
| Bullinger's Lexicon and Concordance to Greek Testament, medium 8vo. | 30s. |
| Collis's Chief Tenses of the Greek Irregular Verbs, 8vo. | 1s. |
| — Pontes Graeci, Stepping Stone to Greek Grammar, 18mo. | 2s. |
| — Praxis Graeca, Etymology, 18mo. | 2s. 6d. |
| — Greek Verse-Book, Praxis Iambica, 18mo. | 4s. 6d. |
| Congreve's Politics of Aristotle, translated, 8vo. | 18s. |
| Cooper's Tales from Euripides, fcp. 8vo. | 2s. 6d. |
| Donaldson's New Cratylus, Fourth Edition, 8vo. | 2s. 6d. |
| — Pindar's Epicnian or Triumphal Odes, 8vo. | 16s. |
| Farrar's Brief Greek Syntax and Accidence, 18mo. | 4s. 6d. |
| — Greek Grammar Rules for Harrow School, 18mo. | 1s. 6d. |
| Fowle's Short and Easy Greek Book, 18mo. | 2s. 6d. |
| — Eton Greek Reading-Book, 18mo. | 1s. 6d. |
| — First Easy Greek Reading-Book, 18mo. | 5s. |
| Grant's Ethics of Aristotle, with Essays and Notes, 2 vols. 8vo. | 32s. |

General Lists of School-Books

- Hawtin's Greek Examination-Papers, 12mo. 1s. 6d.
 Ishbister's Xenophon's *Anabasis*, Books I. & II. with Notes, 12mo. 1s. 6d.
 Kennedy's Greek Grammar, 12mo. 1s. 6d.
 Liddell and Scott's Larger Greek-Lexicon, crown 8vo. 1s.
 — — — Greek-English Lexicon abridged, square 12mo. 1s. 6d.
 Learwoof's *Sophocles*, Greek Text, Latin Notes, 2d Edition, 8vo. 1s.
 — — — Theban Trilogy of Sophocles literally explained, crown 8vo. 1s. 6d.
 Morris's Greek Lessons, square 12mo. Part I. 1s. 6d. Part II. 1s.
 Parry's Elementary Greek Grammar, 12mo. 1s. 6d.
 Cheppard and Evans' Notes on Thucydides, crown 8vo. 1s. 6d.
 Thucydides' Peloponnesian War, translated by Creasy, 8vo. 1s. 6d.
 Tully's Greek Deictica, improved by the Rev. Dr. White, 12mo. 2s. 6d. Key 1s. 6d.
 White's Xenophon's Expedition of Cyrus, with English Notes, 12mo. 1s. 6d.
 Wilkinson's Manual of Greek Prose Composition, crown 8vo. 1s. 6d. Key 1s.
 — — — Exercises in Greek Prose Composition, crown 8vo. 1s. 6d. Key 1s.
 — — — Progressive Greek Deictica, 12mo. 1s. 6d. Key 1s. 6d.
 — — — Progressive Greek Anthology, 12mo. 1s. 6d.
 — — — Scriptores Attici, Extracts with English Notes, crown 8vo. 1s. 6d.
 — — — Speeches from Thucydides translated, part two. 1s.
 Williams's Nicomachean Ethics of Aristotle translated, crown 8vo. 1s. 6d.
 Wright's Plato's *Phaedrus*, *Lysis* and *Protogoras*, translated, 8vo. 1s. 6d.
 Yonge's Larger English-Greek Lexicon, 8vo. 1s.
 — — — English-Greek Lexicon abridged, square 12mo. 1s. 6d.
 Zeller's *Plato and the Older Academy*, by Alleyne & Goodwin, ex. 8vo. 1s.
 — — — Socrates, translated by Reichen, crown 8vo. 1s. 6d.

White's Grammar-School Greek Texts.

- | | |
|---|---|
| <i>Nesop</i> (Fables) and <i>Palestine</i> (Myths), 12mo. Price 1s. | St. Matthew's and St. Luke's Gospels, 1s. 6d. each. |
| <i>Homer</i> , <i>Iliad</i> , Book I. 1s. | St. Mark's and St. John's Gospels, 1s. 6d. each. |
| <i>Locrian</i> , Select Dialogues 1s. | The Acts of the Apostles 1s. 6d. |
| <i>Xenophon</i> , <i>Anabasis</i> , Books I. III. & V. 1s. 6d. each; Book II. 1s. | St. Paul's Epistle to the Romans 1s. 6d. |

The Four Gospels in Greek, with Greek-English Lexicon. Edited by John T. —
 White, D.D. Oxon. Square 12mo. price 5s.

White's Grammar-School Latin Texts.

- | | |
|---|---|
| Cesar, Gallic War, Books I. & II. V. & VI. 1s. each. | Nepos, Miltiades, Cimon, Pausanias, Aristides Price 1s. 6d. |
| Cesar, Gallic War, Books III. & IV. 9d. each. | Ovid, Selections from <i>Epinice</i> and <i>Fasti</i> 1s. |
| Cicero, <i>Cato Major</i> 1s. 6d. | Ovid, Select Myths from Metamorphoses 1s. |
| Cicero, <i>Laelius</i> 1s. 6d. | Phaedrus, Select Easy Fables 1s. |
| Eutropius, Roman History, Books I. & II. 1s. Books III. & IV. 1s. | Phaedrus, Fables, Book I. & II. 1s. |
| Horace, Odes, Book I. II. & IV. 1s. each. | Sallust, Bellum Catilinarium 1s. 6d. |
| Horace, Odes, Book III. 1s. 6d. | Virgil, <i>Georgics</i> , Book IV. 1s. |
| | Virgil, <i>Aeneid</i> , Books I. to VI. each 1s. |

Livy, Books XXII. and XXIII. The Latin Text with English Explanatory and Grammatical Notes, and a Vocabulary of Proper Names. Edited by John T. White, D.D. Oxon. 12mo. price 2s. 6d. each Book.

The Latin Language.

| | | |
|---|---------|---------------|
| Bradley's Latin Prose Exercises, 12mo. | 3s. 5d. | Key 5s. |
| — Continuous Lessons in Latin Prose, 12mo. | 5s. | Key 5s. 6d. |
| — Cornelius Nepos, improved by White, 12mo. | 3s. | 6d. |
| — Ovid's Metamorphoses, improved by White, 12mo. | 4s. | 6d. |
| — Select Fables of Phaedrus, improved by White, 12mo. | 2s. | 6d. |
| — Entropius, improved by White, 12mo. | 2s. | 6d. |
| Cicero's Correspondence, by Tyrrell, VOL. I. 8vo. | 12s. | |
| Collis's Chief Tenses of Latin Irregular Verbs, 8vo. | 1s. | |
| — Ponte's Latin, Stepping Stone to Latin Grammar, 12mo. | 3s. | 6d. |
| Fowle's Short and Easy Latin Book, 12mo. | 1s. | 6d. |
| — First Easy Latin Reading-Book, 12mo. | 3s. | 6d. |
| — Second Easy Latin Reading-Book, 12mo. | 3s. | 6d. |
| Hewitt's Latin Examination-Papers, 12mo. | 1s. | 6d. |
| Ishbister's Caesar, Books I.—VII, 12mo, &c. or with Reading Lessons | 4s. | 6d. |
| — Caesar's Commentaries, Books I.—V, 12mo. | 3s. | 6d. |
| — First Book of Caesar's Gallic War, 12mo. | 1s. | 6d. |
| Jerram's Latina Reddenda, crown 8vo. | 1s. | |
| — and Malan's Anglopitanus | | Nearly ready. |
| Kennedy's Child's Latin Primer, or First Latin Lessons, 12mo. | 1s. | |
| — Child's Latin Accidence, 12mo. | 1s. | |
| — Elementary Latin Grammar, 12mo. | 3s. | 6d. |
| — Elementary Latin Reading-Book or <i>Tirocinium Latinum</i> , 12mo. | 2s. | |
| — Latin Prose, Palaestra Still Latin, 12mo. | 6s. | |
| — <i>Subsidia Primaria</i> . Exercise Books to the <i>Public School Latin Primer</i> , I. Accidence and Simple Construction, 2s. 6d. II. Syntax, 3s. 6d. | | |
| Key to the Exercises in <i>Subsidia Primaria</i> , Parts I. & II. price 5s. | | |
| Kennedy's <i>Subsidia Primaria</i> , III. The Latin Compound Sentence, 12mo... 1s. | | |
| — Curriculum Still Latin, 12mo. 4s. 6d. Key, 7s. 6d. | | |
| — Palaestra Latina, or Second Latin Reading-Book, 12mo. | 5s. | |
| Kenny's Caesar's Commentaries, Book I. 12mo. 1s. Books II. & III. 1s. | | |
| Lewis and Short's Latin Dictionary, 4to. | 31s. | 6d. |
| — Virgil's Aeneid, Books I. II. III. & V. 12mo. each Book 1s. | | |
| Millington's Selections for Latin Prose, crown 8vo. | 3s. 6d. | Key 5s. |
| Moody's Eton Latin Grammar, 12mo. 2s. 6d. The Accidence separately 1s. | | |
| Parry's Origines Romane, from Livy, with English Notes, crown 8vo. 4s. | | |
| The Public School Latin Primer, 12mo. | 2s. | 6d. |
| — Grammar, by Rev. Dr. Kennedy, post 8vo. 7s. 6d. | | |
| Prendergast's Mastery Series, Manual of Latin, 12mo. | 2s. | 6d. |
| Hapier's Introduction to Composition of Latin Verse, 12mo... 3s. 6d. Key 2s. 6d. | | |
| Riddle's Young Scholar's Lat.-Eng. & Eng.-Lat. Dictionary, square 12mo. ... 10s. 6d. | | |
| Separately { The Latin-English Dictionary, 5s. | | |
| Biddle and Arnold's English-Latin Lexicon, 8vo. | 31s. | |
| Sheppard and Turner's Aids to Classical Study, 12mo. 5s. Key 6s. | | |
| Valpy's Latin Delectus, improved by White, 8mo. | 2s. | 6d. |
| Virgil's Works, edited by Kennedy, crown 8vo. | 10s. | 6d. |
| Walford's Progressive Exercises in Latin Elegiac Verse, 12mo. 2s. 6d. Key 5s. | | |
| White and Riddle's Large Latin-English Dictionary, 1 vol. 4to. | 21s. | |
| White's College Latin-English Dictionary (Intermediate size), royal 8vo. 12s. | | |
| White's Junior Student's Eng.-Lat. & Lat.-Eng. Dictionary, sq. 12mo. 12s. | | |
| White's Latin-English Dictionary, price 7s. 6d. | | |
| Separately { The English-Latin Dictionary, price 5s. 6d. | | |
| — Middle-Class Latin Dictionary, square 8vo. 8s. | 3s. | |
| — Cicero's Cato Major and Laelius, 12mo. 3s. 6d. | | |
| Wilkins's Progressive Latin Delectus, 12mo. 2s. | | |
| — Easy Latin Prose Exercises, crown 8vo. 2s. 6d. Key 2s. 6d. | | |
| — Manual of Latin Prose Composition, crown 8vo. 5s. 6d. Key 3s. | | |
| — Latin Prose Exercises, crown 8vo. 4s. 6d. Key 5s. | | |
| — Rules of Latin Syntax, 8vo. 2s. | | |
| — Notes for Latin Lyrics (in use in Harrow, &c.) 12mo. 4s. 6d. | | |
| — Latin Anthology, for the Junior Classes, 12mo. 4s. 6d. | | |
| Xonge's Odes and Epodes of Horace, School Edition, 12mo. 5s. | | |
| — Satires and Epistles of Horace, School Edition, 12mo. 21s. | | |
| — Library Edition of the Works of Horace, 8vo. 12s. | | |
| — Latin Gradua, post 8vo. 2s. or with Appendix | | |

The French Language.

| | |
|--|-----------------------------------|
| Albité's How to Speak French, fop. 8vo..... | 5s. 6d. |
| — Instantaneous French Exercises, fop. 2s. Key, 2s. | |
| Cassal's French Grammars, crown 8vo..... | 3s. 6d. |
| Cassal & Karcher's Graduated French Translation Book, PART I. 2s. 6d. PART II. 3s. | |
| Contanzeau's Practical French and English Dictionary, post 8vo..... | 7s. 6d. |
| Contanzeau's Middle-Class French Course, fop. 8vo. | |
| Accidence, 8d. | French Translation-Book, 8d. |
| Syntax, 8d. | Easy French Delectus, 8d. |
| French Conversation-Book, 8d. | First French Reader, 8d. |
| First French Exercise-Book, 8d. | Second French Reader, 8d. |
| Second French Exercise-Book, 8d. | French and English Dialogues, 8d. |
| Contanzeau's Guide to French Translation, 12mo..... | 3s. 6d. Key 2s. 6d. |
| — Prosateurs et Poëtes Français, 12mo..... | 2s. |
| — Précis de la Littérature Française, 12mo..... | 2s. 6d. |
| — Abrégé de l'Histoire de France, 12mo..... | 3s. 6d. |
| Merlet's French Grammar, fop. 8vo. | |
| French Pronunciation and Accidence, fop. 3s. 6d. | 5s. 6d. } Key, price 3s. 6d. |
| Syntax of the French Grammar, fop. 3s. 6d. | |
| Le Traducteur, fop. 8vo..... | 5s. 6d. |
| Stories for French Writers, fop. 8vo. | 2s. |
| Aperçu de la Littérature Française, fop. 8vo. | 2s. 6d. |
| Exercises in French Composition, fop. 8vo. | 2s. 6d. |
| French Synonyms, fop. 8vo. | 2s. 6d. |
| Synopsis of French Grammar, fop. 8vo. | 2s. 6d. |
| Prendergast's Mastery Series, French, 12mo..... | 2s. 6d. |
| Sewall's Contes Faciles, crown 8vo..... | 2s. 6d. |
| The Stepping-Stone to French Pronunciation, 12mo..... | 1s. |
| Souvestre's Philosophie sous les Toits, by Stévenard, square 12mo..... | 1s. 6d. |
| Stévenard's Lectures Françaises from Modern Authors, 12mo..... | 4s. 6d. |
| — Rules and Exercises on the French Language, 12mo..... | 3s. 6d. |
| Tarver's Eton French Grammar, 12mo..... | 6s. 6d. |

German, Spanish, Hebrew, Sanskrit.

| | |
|---|--|
| Benfey's Sanskrit-English Dictionary, medium 8vo..... | 52s. 6d. |
| Blackley's Practical German & English Dictionary, post 8vo..... | 7s. 6d. |
| Buchheim's German Poetry, for Repetition, 12mo..... | 3s. 6d. |
| Collis's Card of German Irregular Verbs, 8vo..... | 2s. |
| Fischer-Fischart's Elementary German Grammar, fop. 8vo..... | 2s. 6d. |
| Just's German Grammar, 12mo..... | 1s. 6d. |
| — German Reading Book, 12mo..... | 3s. 6d. |
| Kalisch's Hebrew Grammar, 8vo..... | Part I. 12s. 6d. Key 5s. Part II. 12s. 6d. |
| Longman's Pocket German & English Dictionary, square 12mo..... | 5s. |
| Milne's Practical Mnemonic German Grammar, crown 8vo..... | 2s. 6d. |
| Müller's (Max) Sanskrit Grammar for Beginners, royal 8vo..... | 15s. |
| Nafel's Elementary German Course for Public Schools, fop. 8vo. | |
| German Accidence, 9d. | German Prose Composition Book, 9d. |
| German Syntax, 9d. | First German Reader, 9d. |
| First German Exercise-Book, 9d. | Second German Reader, 9d. |
| Second German Exercise-Book, 9d. | |
| Prendergast's Handbook to the Mastery Series, 12mo..... | 2s. |
| — Mastery Series, German, 12mo..... | 2s. 6d. |
| — Manual of Spanish, 12mo..... | 2s. 6d. |
| — Manual of Hebrew, crown 8vo..... | 2s. 6d. |
| Wirth's German Chit-Chat, crown 8vo..... | 2s. 6d. |

London, LONGMANS & CO.!





